

## 2. Preliminary

We present preliminary analysis of the time-based admission control without buffering requests in an idealized transmission environment. The purpose is to illustrate the spirit of this study and some limitations of underlying system parameters. In Section 2.1, we outline the subject of the chapter. The data rate function adopted in analysis is presented in Section 2.2. In Section 2.3, we define a measure of performance improvement and then present analysis for the measure. Numerical results from the analysis are discussed in Section 2.4.

### 2.1 Preliminary

We consider an ideal wireless environment where data over forward link can be transmitted at any rate according to a continuous rate function. The rate function is assumed to be deterministic depending on distance-related path-loss exponent and the distance from BST to a target MT. Under this environment and the condition of system stability, our primary focus is the factor of improvement in serviced request rates; that is, the ratio of the rate of serviced requests when the forward-link data service system is under time-based admission control to that when the system is operated without admission control. Although the forward link transmission system is considerably simplified, it is enough for us to look at the impact of underlying system parameters.

## 2.2 Data Rate Function

A standard model for peak rate function is

$$C = \frac{W}{E_b / N_o} SINR, \quad (2.1)$$

where  $E_b$  is the energy per information bit,  $N_o$  the total interference and noise power spectrum density,  $W$  the forward link bandwidth,  $SINR$  the signal to interference and noise power ratio at a target MT, and  $C$  the data rate for the MT [15]. When the spectral efficiency in bits/sec/Hz is less than one,  $E_b/N_o$  can be regarded as a constant for all data rate, according to Shannon's capacity limit in AWGN channels. Thus, the data rate for the MT is proportional to its received  $SINR$ , which is essentially a random variable depending on distance-related path-loss, fading, shadowing, and interference power. For a high data rate system [15], it is more appropriate to assume the rate function  $C = f(SINR)$ , where  $f$  is typically expressed as

$$f(SINR) = W \log_2(1 + \eta SINR), 0 \leq \eta < 1, \quad (2.2)$$

limited by the Shannon capacity function and modem performance  $\eta$ . However, we adopt a deterministic peak rate function [9],

$$C(r) = C_0 \times \begin{cases} 1 & , \text{if } r \leq r_0 \\ \left(\frac{r_0}{r}\right)^\alpha & \text{otherwise.} \end{cases} \quad (2.3)$$

where  $r$  is the distance from the BST to a target MT,  $C_0$  the maximum data rate that can be achieved,  $\alpha$  the path-loss exponent, and  $r_0$  the maximum distance at which the

MT can receive  $C_0$  data rate. The path-loss component typically has a value between 2 and 5. Note that intercell interference, fading, and shadowing effects are not considered in rate function  $C(r)$ .

## 2.3 Performance Analysis

Let  $\Gamma$  be a time threshold used as a criterion for admission of forward link data traffics. That is, if a service request for air time resources is larger (resp. smaller) than  $\Gamma$ , it is blocked (resp. accepted). Traffic arrives in batch. Let  $V$  be a random variable representing the volume (size), in bits, of a request. Let  $\sigma = E[V]$ , the expected batch size. Assume that the distribution of batch size is independent of MT's spatial location. Then, the air time resource required to service a request for a target MT at a distance  $r$  from the BST is  $V/C(r)$ . The probability of blocking the service request, denoted by  $b(r, \Gamma)$ , is

$$b(r, \sigma, \Gamma) = \Pr \left\{ \frac{V}{C(r)} > \Gamma \right\}. \quad (2.4)$$

Assume that arrivals of service requests are uniformly distributed in two dimensional space with rate  $\lambda$  per unit area. We use  $\Lambda(\lambda, \sigma, \Gamma)$  to indicate the total rate of admitted service requests. Then, we have

$$\Lambda(\lambda, \sigma, \Gamma) = \int_0^R \lambda 2\pi r dr (1 - b(r, \sigma, \Gamma)), \quad (2.5)$$

where  $R$  is the radius of a cell.

On the other hand, the total admitted traffic load in terms of required service time, represented by  $\rho(\lambda, \sigma, \Gamma)$ , can be computed by

$$\rho(\lambda, \sigma, \Gamma) = \int_0^R \lambda 2\pi r dr \int_0^\infty z dF_{\frac{V}{C(r)} | \{\frac{V}{C(r)} < \Gamma\}}(z) (1 - b(r, \sigma, \Gamma)), \quad (2.6)$$

where  $F_{\frac{V}{C(r)} | \{\frac{V}{C(r)} < \Gamma\}}(z)$  is the conditional cumulative distribution function of requested air time  $V/C(r)$ .

We can assume that the forward-link service system is a work-conservative server with service rates depending on the MT in service. Our objective is to maximize  $\Lambda(\lambda, \sigma, \Gamma)$  subject to the constraint of system stability, which requires admitted traffic load  $\rho(\lambda, \sigma, \Gamma) < 1$ . In order to see the effect of time-based admission control, we define  $\psi(\sigma, \Gamma)$  as the factor of improvement in the rate of serviced request under the stability constraint; that is,

$$\phi(\sigma, \Gamma) = \frac{\Lambda^*(\lambda, \sigma, \Gamma)}{\Lambda^*(\lambda, \sigma, \Gamma \rightarrow \infty)}, \quad (2.7)$$

where  $\Lambda^*$  indicates the maximum rate of (2.5) subject to the stability constraint. Note that  $\Lambda^*(\lambda, \sigma, \Gamma \rightarrow \infty)$  is the maximum rate of serviced requests, maximum departure rate, when the system is operated without admission control.

To compute  $\Lambda^*(\lambda, \sigma, \Gamma \rightarrow \infty)$ , we first find the largest possible value of  $\lambda$  by letting  $\rho(\lambda, \sigma, \Gamma \rightarrow \infty) < 1$  in (2.6) and then use the maximum  $\lambda$  in (2.5). We thus have

$$\Lambda^*(\lambda, \sigma, \Gamma \rightarrow \infty) = \left( \sigma \int_0^R \frac{2rdr}{R^2 C(r)} \right)^{-1}. \quad (2.8)$$

In fact, the term  $\left( \sigma \int_0^R \frac{2rdr}{R^2 C(r)} \right)$  in (2.8) is the mean air time required for the system without admission control to service a request.

To find the factor  $\psi$  in (2.7), we repeat the above steps, by first obtaining the largest possible value of  $\lambda$  from (2.6) and substituting the result for  $\lambda$  in (2.5), and then divide (2.5) by (2.8). We thus obtain

$$\phi(\sigma, \Gamma) = \frac{\int_0^R 2\pi r dr (1 - b(r, \sigma, \Gamma)) \sigma \int_0^R \frac{2rdr}{R^2 C(r)}}{\int_0^R 2\pi r dr \int_0^\infty z dF_{\frac{V}{C(r)} | \{ \frac{V}{C(r)} < \Gamma \}}(z) (1 - b(r, \sigma, \Gamma))}. \quad (2.9)$$

Given cell radius  $R$ , the average blocking probability, denoted by  $b(\sigma, \Gamma)$ , is

$$b(\sigma, \Gamma) = \frac{\int_0^R 2\pi r dr b(r, \sigma, \Gamma)}{\pi R^2}. \quad (2.10)$$

Assume that the request size  $V$  is exponentially distributed with mean  $\sigma$ . Then, the expressions for (2.9) and (2.10), respectively, become

$$\phi(\sigma, \Gamma) = \frac{\int_0^R 2rdr(1 - e^{-\frac{C(r)\Gamma}{\sigma}}) \sigma \int_0^R \frac{2rdr}{R^2 C(r)}}{\int_0^R 2rdr \frac{\sigma}{C(r)} \left[ 1 - e^{-\frac{C(r)\Gamma}{\sigma}} \left( 1 + \frac{C(r)}{\sigma} \Gamma \right) \right]} \quad (2.11)$$

and

$$b(\sigma, \Gamma) = \frac{\int_0^R 2rdr e^{-\frac{C(r)\Gamma}{\sigma}}}{R^2}. \quad (2.12)$$

## 2.4 The Effect of Underlying System Parameters

To investigate the effects of varying admission time-threshold on the improvement factor of serviced request rate  $\psi(\sigma, \Gamma)$  and on average blocking probability  $b(\sigma, \Gamma)$ , we set normalized close-in radius  $r_0=1$ , path-loss exponent  $\sigma=2$ , maximum peak service rate  $C_0=2457.6\text{Kbps}$ , and consider two mean request sizes,  $\sigma=81920$  bits and  $\sigma=40960$  bits, and three possible cell coverage radii,  $R=2, 3, 4$ . Numerical data for (2.11) and (2.12) are illustrated in Figs. 2-1 (a) and (b), respectively. It can be seen that the improvement factor and average blocking probability both decrease with admission time threshold. In order to obtain a high improvement factor, the admission time threshold must be small and service requests arrive at high rate. This also gives rise to a very high average blocking probability, which implies that the admission control is very selective and thus not feasible. Considering the random fluctuation of wireless forward link data loads, it is however possible to significantly reduce average blocking probabilities by exploiting high improvement factors and

buffering long air-time service requests, instead of dropping them, when traffic arrival rates are high temporally or spatially. This is what we will study later in the thesis.

Comparing results for  $\sigma=81920$  bits and  $\sigma = 40960$  bits in Figs 2-1 (a) and (b), we see that arrival traffics with larger mean data sizes suffer more blocking probabilities but have more potential for obtaining higher improvement factors. In fact, they provide more chances for admission controller to discriminate against services for long air-time requests. Comparing results for  $R=2,3$ , and 4 in Figs 2-1 (a) and (b), we also see that the effect of large cell coverage is similar to that of larger request size distribution, because of lower service rate and hence longer service air-time requirement for MTs at far field. Therefore, large cell coverage or large request size distribution gives rise to more dynamic service air-time requirements, which more or less imposes the requirement of queueing service requests on using time-based admission control.

As to the effects of varying path-loss exponents, we set  $r_0=1$ ,  $R=3$ ,  $C_0=2457.6$ Kbps, and consider two mean request sizes,  $\sigma=81920$  bits and  $\sigma =40960$  bits, and three possible path-loss exponents,  $\alpha=2, 3, 4$ . Numerical results for (2.11) and (2.12) are illustrated in Figs. 2-2 (a) and (b), respectively. It can be seen that both the

improvement factor and average blocking probability increase with path-loss exponent  $\alpha$ . In fact, the effect of larger  $\alpha$  is lower peak service data rate and longer service air time requirement, similar to the effect of increasing cell coverage discussed previously.

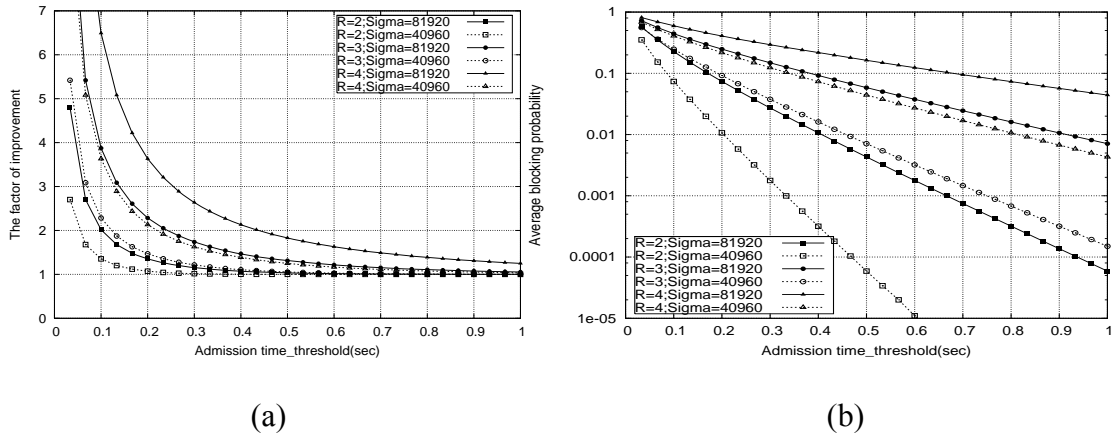
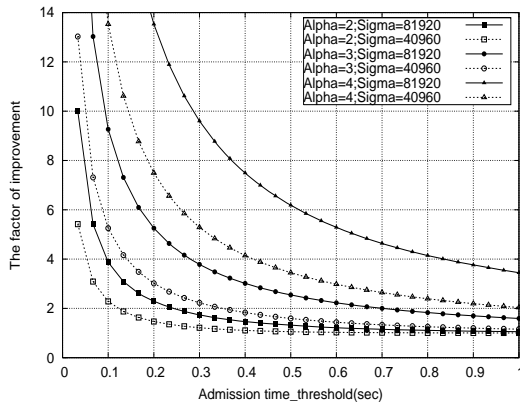
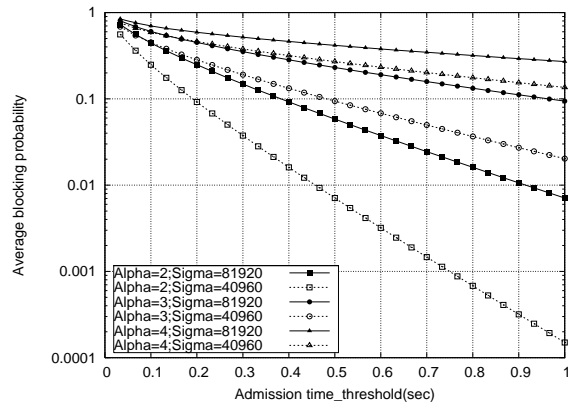


Figure 2. 1 (a) The factor of improvement in serviced request rate  $\psi(\sigma,\Gamma)$  versus admission time threshold  $\Gamma$  (second); (b) average blocking probability  $b(\sigma,\Gamma)$  versus admission time threshold  $\Gamma$  (second), for normalized close-in radius  $r_0=1$ , cell coverage radii  $R=2,3,4$ , path-loss exponent  $\alpha=2$ , and maximum peak service rate  $C_0=2457.6\text{Kbps}$ .





(a)



(b)

Figure 2. 2 (a) The factor of improvement in serviced request rate  $\psi(\sigma, \Gamma)$  versus admission time threshold  $\Gamma$  (second); (b) average blocking probability  $b(\sigma, \Gamma)$  versus admission time threshold  $\Gamma$  (second), for normalized close-in radius  $r_0=1$ , cell coverage radius  $R=3$ , path-loss exponent  $\alpha=2, 3, 4$ , and maximum peak service rate  $C_0=2457.6\text{Kbps}$