4. Simulation Studies

We conducted simulation studies to examine the finite sample properties of the proposed three confidence intervals for $\rho^{(1)} - \rho^{(2)}$. All the simulations are based on 1000 simulated datasets. The following notation will be used in the presentation of the simulation results.

$\rho^{(1)}$: the correlation coefficient from bivariate normal distribution 1.

$\rho^{(2)}$: the correlation coefficient from bivariate normal distribution 2.

n_1 : the size of sample 1.

n_2 : the size of sample 2.

CPI: coverage probability of confidence interval I.

CPII: coverage probability of confidence interval II.

CPIII: coverage probability of confidence interval III.

LI: mean of the length of $100(1-\alpha)\%$ C.I. of $\left|\rho^{(1)} - \rho^{(2)}\right|$ based on confidence interval I.

LII: mean of the length of $100(1-\alpha)\%$ C.I. of $\left|\rho^{(1)} - \rho^{(2)}\right|$ based on confidence interval II.

LIII: mean of the length of $100(1-\alpha)\%$ C.I. of $\left|\rho^{(1)} - \rho^{(2)}\right|$ based on confidence interval III.

PI: (times of CI(II) is wider than CI(III))/1000.

PII: (times of CI(I) is wider than CI(III))/1000.

PIII: (times of CI(I) is wider than CI(II))/1000.

Table 1(a). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0.7, 0.8, 0.05)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	1.0	0.963	0.931	1.607	0.466	0.388	1.0	1.0	1.0
(30, 90)	1.0	0.935	0.903	1.129	0.402	0.324	1.0	1.0	1.0
(90, 30)	1.0	0.951	0.942	1.351	0.344	0.325	1.0	1.0	1.0
(100, 100)	1.0	0.945	0.923	0.904	0.248	0.225	1.0	1.0	1.0
(100, 200)	1.0	0.955	0.933	0.772	0.224	0.195	1.0	1.0	1.0
(200, 100)	1.0	0.946	0.942	0.793	0.201	0.196	1.0	1.0	1.0
(300, 300)	1.0	0.935	0.940	0.523	0.141	0.133	1.0	1.0	1.0
(300, 400)	1.0	0.951	0.933	0.486	0.136	0.124	1.0	1.0	1.0
(400, 300)	1.0	0.959	0.950	0.492	0.129	0.124	1.0	1.0	1.0

Table 1(b). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0.7, 0.8, 0.1)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	1.0	0.908	0.862	1.344	0.390	0.331	1.0	1.0	1.0
(30, 90)	0.999	0.877	0.825	1.083	0.336	0.275	0.957	1.0	1.0
(90, 30)	1.0	0.894	0.893	1.130	0.288	0.276	0.828	1.0	1.0
(100, 100)	1.0	0.889	0.871	0.756	0.206	0.189	1.0	1.0	1.0
(100, 200)	1.0	0.909	0.854	0.646	0.187	0.164	0.992	1.0	1.0
(200, 100)	1.0	0.890	0.885	0.663	0.168	0.165	0.793	1.0	1.0
(300, 300)	1.0	0.911	0.886	0.438	0.118	0.110	1.0	1.0	1.0
(300, 400)	1.0	0.903	0.872	0.407	0.114	0.104	1.0	1.0	1.0
(400, 300)	1.0	0.895	0.883	0.411	0.108	0.104	1.0	1.0	1.0

Table 2(a). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0.4, 0.7, 0.05)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.999	0.945	0.899	1.379	0.713	0.564	1.0	1.0	1.0
(30, 90)	0.996	0.919	0.830	1.068	0.633	0.469	1.0	1.0	1.0
(90, 30)	0.999	0.935	0.926	1.200	0.512	0.472	1.0	1.0	1.0
(100, 100)	1.0	0.935	0.900	0.775	0.385	0.324	1.0	1.0	1.0
(100, 200)	0.998	0.951	0.886	0.642	0.357	0.281	1.0	1.0	1.0
(200, 100)	1.0	0.943	0.927	0.697	0.308	0.281	1.0	1.0	1.0
(300, 300)	1.0	0.955	0.912	0.448	0.222	0.189	1.0	1.0	1.0
(300, 400)	1.0	0.954	0.909	0.411	0.215	0.177	1.0	1.0	1.0
(400, 300)	1.0	0.960	0.926	0.426	0.201	0.178	1.0	1.0	1.0

Table 2(b). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0.4, 0.7, 0.1)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.995	0.894	0.816	1.154	0.597	0.477	1.0	1.0	0.998
(30, 90)	0.982	0.850	0.752	0.894	0.530	0.396	1.0	1.0	1.0
(90, 30)	0.994	0.892	0.879	1.00	0.428	0.398	1.0	1.0	1.0
(100, 100)	1.0	0.889	0.828	0.648	0.322	0.272	1.0	1.0	1.0
(100, 200)	0.994	0.901	0.802	0.537	0.299	0.235	1.0	1.0	1.0
(200, 100)	0.999	0.893	0.868	0.583	0.258	0.236	1.0	1.0	1.0
(300, 300)	1.0	0.905	0.843	0.375	0.186	0.158	1.0	1.0	1.0
(300, 400)	0.995	0.913	0.842	0.344	0.180	0.148	1.0	1.0	1.0
(400, 300)	0.999	0.899	0.841	0.356	0.168	0.149	1.0	1.0	1.0

Table 3(a). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0.3, 0.3, 0.05)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.939	0.943	0.980	1.126	0.916	0.821	1.0	0.961	0.874
(30, 90)	0.917	0.930	0.975	0.922	0.745	0.694	1.0	0.954	0.862
(100, 100)	0.937	0.938	0.984	0.624	0.503	0.485	1.0	0.998	0.987
(100, 200)	0.944	0.949	0.980	0.539	0.435	0.424	1.0	1.0	1.0
(300, 300)	0.949	0.949	0.988	0.360	0.290	0.287	1.0	1.0	1.0
(300, 400)	0.956	0.956	0.984	0.336	0.272	0.269	1.0	1.0	1.0

Table 3(b). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0.3, 0.3, 0.1)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.952	0.876	0.885	0.942	0.767	0.701	1.0	0.946	0.874
(30, 90)	0.939	0.859	0.872	0.771	0.623	0.589	1.00	0.943	0.862
(100, 100)	0.946	0.887	0.887	0.522	0.420	0.409	1.0	0.997	0.987
(100, 200)	0.956	0.881	0.883	0.451	0.364	0.357	1.0	0.994	0.987
(300, 300)	0.962	0.900	0.902	0.301	0.243	0.241	1.0	1.0	1.0
(300, 400)	0.958	0.903	0.903	0.281	0.228	0.226	1.0	1.0	1.0

Table 4(a). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0, 0.2, 0.05)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.969	0.923	0.929	1.045	0.976	0.868	1.0	0.880	0.645
(30, 90)	0.956	0.920	0.927	0.843	0.805	0.737	1.0	0.806	0.598
(90, 30)	0.953	0.926	0.940	0.862	0.789	0.734	1.0	0.802	0.635
(100, 100)	0.956	0.941	0.939	0.571	0.541	0.514	1.0	0.875	0.735
(100, 200)	0.956	0.938	0.933	0.489	0.471	0.448	1.0	0.829	0.677
(200, 100)	0.956	0.938	0.933	0.489	0.471	0.448	1.0	0.829	0.677
(300, 300)	0.963	0.944	0.941	0.329	0.313	0.303	1.0	0.932	0.858
(300, 400)	0.963	0.950	0.943	0.306	0.293	0.284	1.0	0.934	0.844
(400, 300)	0.957	0.943	0.939	0.308	0.292	0.284	1.0	0.931	0.854

Table 4(b). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (0, 0.2, 0.1)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.917	0.872	0.872	0.874	0.817	0.739	1.0	0.837	0.645
(30, 90)	0.900	0.854	0.856	0.705	0.674	0.624	1.0	0.782	0.598
(90, 30)	0.899	0.865	0.877	0.721	0.660	0.622	1.0	0.773	0.635
(100, 100)	0.905	0.883	0.881	0.477	0.452	0.433	1.0	0.851	0.735
(100, 200)	0.889	0.876	0.872	0.409	0.394	0.377	1.0	0.810	0.677
(200, 100)	0.916	0.889	0.888	0.417	0.389	0.376	1.0	0.846	0.767
(300, 300)	0.917	0.900	0.896	0.27	0.262	0.254	1.0	0.926	0.858
(300, 400)	0.909	0.902	0.893	0.256	0.245	0.238	1.0	0.933	0.844
(400, 300)	0.907	0.888	0.883	0.258	0.244	0.238	1.0	0.923	0.854

Table 5(a). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (-0.2, -0.3, 0.05)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.971	0.929	0.932	1.105	0.932	0.833	1.0	0.949	0.833
(30, 90)	0.971	0.924	0.929	0.888	0.773	0.707	1.0	0.921	0.766
(90, 30)	0.967	0.925	0.935	0.905	0.756	0.710	1.0	0.902	0.812
(100, 100)	0.979	0.948	0.949	0.602	0.517	0.496	1.0	0.977	0.940
(100, 200)	0.971	0.942	0.942	0.517	0.451	0.433	1.0	0.973	0.935
(200, 100)	0.974	0.944	0.945	0.526	0.444	0.434	1.0	0.981	0.960
(300, 300)	0.977	0.941	0.941	0.349	0.299	0.293	1.0	0.999	0.9980
(300, 400)	0.980	0.948	0.946	0.325	0.280	0.275	1.0	0.999	0.999
(400, 300)	0.974	0.941	0.942	0.328	0.278	0.274	1.0	0.999	0.997

Table 5(b). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (-0.2, -0.3, 0.1)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.928	0.871	0.876	0.925	0.780	0.710	1.0	0.924	0.833
(30, 90)	0.930	0.869	0.871	0.743	0.646	0.600	1.0	0.905	0.766
(90, 30)	0.930	0.870	0.886	0.757	0.632	0.602	1.0	0.887	0.812
(100, 100)	0.946	0.898	0.897	0.504	0.432	0.418	1.0	0.972	0.940
(100, 200)	0.929	0.886	0.884	0.433	0.377	0.364	1.0	0.966	0.935
(200, 100)	0.944	0.899	0.903	0.440	0.371	0.365	1.0	0.977	0.960
(300, 300)	0.936	0.887	0.885	0.292	0.250	0.246	1.0	0.999	0.998
(300, 400)	0.941	0.887	0.884	0.272	0.234	0.230	1.0	0.999	0.999
(400, 300)	0.938	0.884	0.884	0.274	0.233	0.230	1.0	0.998	0.997

Table 6(a). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (-0.4, -0.7, 0.05)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	1.0	0.935	0.888	1.391	0.702	0.556	1.0	1.0	1.0
(30, 90)	0.991	0.919	0.845	1.070	0.634	0.465	1.0	1.0	0.999
(90, 30)	1.0	0.947	0.935	1.195	0.513	0.473	1.0	1.0	1.0
(100, 100)	1.0	0.939	0.907	0.770	0.386	0.323	1.0	1.0	1.0
(100, 200)	0.999	0.935	0.872	0.641	0.357	0.280	1.0	1.0	1.0
(200, 100)	1.0	0.942	0.926	0.695	0.308	0.282	1.0	1.0	1.0
(300, 300)	0.999	0.940	0.894	0.447	0.222	0.188	1.0	1.0	1.0
(300, 400)	1.0	0.943	0.884	0.412	0.214	0.177	1.0	1.0	1.0
(400, 300)	1.0	0.947	0.902	0.426	0.201	0.177	1.0	1.0	1.0

Table 6(b). $\left(\rho^{(1)}, \rho^{(2)}, \alpha\right) = (-0.4, -0.7, 0.1)$

(n_1, n_2)	CPI	CPII	CPIII	LI	LII	LIII	PI	PII	PIII
(30, 30)	0.991	0.878	0.813	1.164	0.588	0.471	1.0	1.0	1.0
(30, 90)	0.979	0.860	0.770	0.895	0.531	0.392	1.0	1.0	0.999
(90, 30)	0.997	0.889	0.869	1.0	0.429	0.399	1.0	1.0	1.0
(100, 100)	0.999	0.89	0.841	0.644	0.323	0.271	1.0	1.0	1.0
(100, 200)	0.994	0.885	0.797	0.536	0.299	0.235	1.0	1.0	1.0
(200, 100)	1.00	0.901	0.878	0.581	0.258	0.237	1.0	1.0	1.0
(300, 300)	0.997	0.887	0.836	0.374	0.186	0.158	1.0	1.0	1.0
(300, 400)	0.998	0.893	0.816	0.344	0.179	0.148	1.0	1.0	1.0
(400, 300)	1.0	0.888	0.844	0.356	0.168	0.148	1.0	1.0	1.0

Our simulation studies reveal the following:

1. All the three confidence intervals described above have correct coverage probabilities (CPs) in most of the situations we considered.

2. The confidence interval I tends to have CPs higher than norminal values when both the two population correlations are large in magnitude.

3. The confidence interval III tends to have CPs lower than norminal values when one of the two population correlations is large in magnitude and the sample sizes are highly imbalanced among the two samples.

4. Regarding the precision (length) of the confidence interval, the length of confidence interval III is almost always shorter than that of the confidence interval II, and the length of confidence interval II tends to be shorter than that of confidence interval I. Therefore, for practical applications, we would suggest using the confidence interval III when the two correlations are moderate in magnitude or when the two sample sizes are balenced, while in other situations the confidence interval II is a better choice.