3 Classification of solutions

Recall the problem

$$
(3.1) \qquad \begin{cases} v' = \kappa^2 \\ \kappa' = -\kappa v - \omega \\ v(0) = 0, \kappa(0) = \kappa_0 \end{cases}
$$

By the previous results, we can classify the solutions of (3.1) into the following types.

Definition 1

(A) The solution $(v(s), \kappa(s))$ of (3.1) stays in the region $\{\kappa < 0\}$ for all $s > 0$ and $(v(s), \kappa(s)) \to (+\infty, 0)$ as $s \to +\infty$.

(B) There exists $s_0 > 0$ such that the solution $(v(s), \kappa(s))$ of (3.1) stays in the region $\{\kappa < 0\}$ for all $s \in (s_0, +\infty)$, $\kappa(s) > 0$ for all $s \in (0, s_0)$ and $(v(s), \kappa(s)) \to (+\infty, 0)$ as $s \to +\infty$.

(C) The solution $(v(s), \kappa(s))$ of (3.1) stays in the region $\{\kappa > 0\}$ for all $s > 0$ and $(v(s), \kappa(s)) \to (+\infty, 0)$ as $s \to +\infty$.

(D) There exists $s_0 > 0$ such that the solution $(v(s), \kappa(s))$ of (3.1) stays in the region $\{\kappa > 0\}$ for all $s \in (s_0, +\infty)$, $\kappa(s) < 0$ for all $s \in (0, s_0)$ and $(v(s), \kappa(s)) \to (+\infty, 0)$ as $s \to +\infty$.

(E) The solution $(v(s), \kappa(s))$ of (3.1) stays in the region $\{\kappa > 0\}$ for all $s > 0$ and $(v(s), \kappa(s)) \to (|\kappa_0|, 0)$ as $s \to +\infty$.

(F) The solution $(v(s), \kappa(s))$ of (3.1) stays in the region $\{\kappa < 0\}$ for all $s > 0$ and $(v(s), \kappa(s)) \to (|\kappa_0|, 0)$ as $s \to +\infty$.

Then the following theorems can be deduced from the results in §2.

Theorem 1 *Given $\omega > 0$. Then $(v(s), \kappa(s))$ is of type (A), if $\kappa_0 \leq 0$, and $(v(s), \kappa(s))$ is of type (B), if $\kappa_0 > 0$.*

Theorem 2 *Given $\omega < 0$. Then $(v(s), \kappa(s))$ is of type (C), if $\kappa_0 \geq 0$, and $(v(s), \kappa(s))$ is of type (D), if $\kappa_0 < 0$.*

Theorem 3 *Let $\omega = 0$. Then $(v(s), \kappa(s))$ is of type (E), if $\kappa_0 > 0$; $(v(s), \kappa(s))$ is of type (F), if $\kappa_0 < 0$; and $v(s) = \kappa(s) = 0$ for all $s \geq 0$, when $\kappa_0 = 0$.*