5 Code of the graphics

In this section, we give the code of the graphics by Mathematica for your reference.

5.1 The case for $\omega > 0$

$$\begin{aligned}
\text{In[1]} :=\ & \text{A} = \text{NDSolve}[\{v'[t] == (k[t])^2, k'[t] == -k[t]v[t] - 1,\\
& k[0] == 2, v[0] == 0\}, \{v, k\}, \{t, 0, 10\}]
\end{aligned}$$

$$\begin{aligned}
\text{In[2]} :=\ & \text{B} = \text{NDSolve}[\{v'[t] == (k[t])^2, k'[t] == -k[t]v[t] - 1,\\
& k[0] == -1, v[0] == 0\}, \{v, k\}, \{t, 0, 10\}]
\end{aligned}$$

$$\begin{aligned}
\text{In[3]} :=\ & \text{O} = \text{ParametricPlot}[\{Cos[t], Sin[t]\}, \{t, 0, 2 * \text{Pi}\},\\
& \text{PlotStyle} \to \text{Dashing}[\{0.02, 0.02\}], \text{PlotPoints} \to 1000,\\
& \text{DisplayFunction} \to \text{Identity}]
\end{aligned}$$

$$\begin{aligned}
\text{In[4]} :=\ & \text{P1} = \text{ParametricPlot}[\text{Evaluate}[\{1 - \int_0^t Cos[\int_0^x -k[s]ds]dx,\\
& \int_0^t Sin[\int_0^x -k[s]ds]dx\}/.\text{A}], \{t, 0, 20\}, \text{PlotRange} \to \text{All},\\
& \text{PlotPoints} \to 1000, \text{DisplayFunction} \to \text{Identity}]
\end{aligned}$$

$$\begin{aligned}
\text{In[5]} :=\ & \text{P2} = \text{ParametricPlot}[\text{Evaluate}[\{\int_0^t Cos[\int_0^x -k[s]ds]dx + 1,\\
& \int_0^t Sin[\int_0^x -k[s]ds]dx\}/.\text{B}], \{t, 0, 20\}, \text{PlotRange} \to \text{All},\\
& \text{PlotPoints} \to 1000, \text{DisplayFunction} \to \text{Identity}]
\end{aligned}$$

$$\begin{aligned}
\text{In[6]} :=\ & \text{Show}[\text{P1}, \text{O}, \text{AspectRatio} \to \text{Automatic},\\
& \text{Axes} \to \text{False}, \text{DisplayFunction} \to \$\text{DisplayFunction}]
\end{aligned}$$

$$\begin{aligned}
\text{In[7]} :=\ & \text{Show}[\text{P2}, \text{O}, \text{AspectRatio} \to \text{Automatic},\\
& \text{Axes} \to \text{False}, \text{DisplayFunction} \to \$\text{DisplayFunction}]
\end{aligned}$$

Then we can draw Figure 1 and Figure 2. Note that they are the numerical solutions.

5.2 The case for $\omega = 0$

In particular, we can solve the solution of the case for $\omega = 0$ by Mathematica.

$$\begin{aligned}
\text{In[1]} :=\ & \text{DSolve}[\{v'[t] == (k[t])^2, k'[t] == -k[t]v[t],\\
& k[0] == 1, v[0] == 0\}, \{v[t], k[t]\}, t]
\end{aligned}$$

$$\text{Out[1]} :=\ \{\{k[t] \to \sqrt{(1 - Tanh[t]^2)}, v[t] \to Tanh[t]\}\}$$

Then we can compute the curve by the formula (4.1) and (4.2) and we draw Figure 3.