1 Introduction

In this paper, we study the steadily rotating spiral wave in the kinematic theory of the excitable media (cf. [6, 1, 5, 4, 2]). Let s be the arc length measured from the tip, $\kappa := \kappa(s)$ be the curvature of the curve, $u := u(s)$ be the normal velocity and ω is the constant angular frequency when $\omega \neq 0$. Moreover, we choose the normal vector to be the left-hand normal to the tangent vector and the curvature to be positive when the curve is winding in the clockwise direction.

We study the integro-differential equation

$$u'(s) + \kappa(s) \int_0^s \kappa(\xi)u(\xi)d\xi = \omega, s \geq 0, \tag{1.1}$$

under the curvature flow

$$u = -\kappa. \tag{1.2}$$

Then (1.1) is reduced to

$$-\kappa'(s) - \kappa(s) \int_0^s \kappa^2(\xi)d\xi = \omega, s \geq 0. \tag{1.3}$$

It is important to note that if $\omega \neq 0$ the radius of the core circle is given by $\rho = \dfrac{|u(0)|}{|\omega|}$ with the tangent of the tip pointing inward to the center of the core circle if $u(0) < 0$; outward to the center if $u(0) > 0$ (cf. [3]).

Now, we set

$$v(s) := \int_0^s \kappa^2(\xi)d\xi, s > 0. \tag{1.4}$$

Then (1.3) is reduced to the system

$$\begin{cases} v' = \kappa^2 \\ \kappa' = -\kappa v - \omega, \end{cases} \tag{1.5}$$

and the initial condition for (1.5) shall be given by

$$v(0) = 0, \kappa(0) = \kappa_0, \tag{1.6}$$

where κ_0 is the curvature at the tip.

We shall study the initial value problem for the system (1.5) with the initial condition (1.6) for all ω and κ_0. We classify all solutions in all cases.

This paper is organized as follows. In §2, by the phase plane analysis, we derives the global existence of solutions and study the asymptotically behaviors of these solutions. We classify in §3 the solutions by using the tip curvature κ_0 and ω as parameters. Then, in §4, we give some geometric explanations of the results obtained in §2. Finally, the codes of graphics by Mathematica are given in §5 for the readers' convenience.