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Supersymmetry Breaking by Metastable vacua

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To my parents

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Supersymmetry Breaking by Metastable Vacua

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ABSTRACT:
We study supersymmetry breaking by metastable vacua. First we investigate the general O'Raiferataigh models. These models use a non-zero F-term to break SUSY. We show how an additional small interaction term would generate a SUSY vacuum and a metastable SUSY breaking state. The general features are studied when we extend the pseudo-potential to a more general form and we find some very unusual moduli structures in certain interesting choices. Then we consider the work of K.R. Dienes and B. Thomas (2008). In this latter model, both D-term and F-term are present in the scalar potential. Hence it offers a very rich vacuum structure including several unstable states, a SUSY vacuum and a metastable SUSY breaking state. We generalize the model by various simplifications and demonstrate how the crucial features could be preserved and even improved upon. In both of the above cases, the vacuum structure can be determined perturbatively at tree level. This is a huge advantage over strong interacting SUSY breaking models. The tunneling between the metastable state and the real vacuum can be calculated. We study some physical implications of these models.

KEYWORDS: Supersymmetry breaking, O'Raifeartaigh model, Metastable vacua.

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Contents

1. Introduction

The Standard Model (SM) of particle physics can describe almost all the basic forces from the electroweak interaction to the nuclear interaction. However, there are still intrinsic problems in Standard Model, including the hierarchy, naturalness and triviality problems. These problems indicate Standard Model can only be viewed as an effective theory and there must be new things to be discovered at TeV scale by The Large Hadron Collider (LHC).

Supersymmetry (SUSY) is one of the most acceptable extension beyond Standard Model that emerges at high energy to solve the hierarchy problem. The symmetry relates bosons and fermions. Nevertheless, the real world particles are not degenerate with their superpartners. Therefore supersymmetry must be broken. Supersymmetry breaking is unlike other internal symmetry breaking in ordinary quantum field theory (QFT). Supersymmetry provides rigorous restrictions on the theory. For example, the property of holomorphicity of the superpotential prohibits its coefficients to obtain any renormalization. These restrictions are so strong that we can sometimes arrive at exact non-perturbative results. However, there is always a dark side beneath the bright surface; holomorphicity also imposes very strict constraint on the theory so that breaking SUSY becomes very difficult and non-generic. One of the strict limits comes from the Witten index[1], which is a robust property and won't change under normal changes of the Lagrangian. It was shown that Witten index could not be zero if SUSY is broken. This rules out all SUSY Yang-Mills theory with vector-like matter from SUSY breaking. This is an indication that the vacuum of a SUSY theory has topological nature: it depends only on the asymptotics and the global property of the theory. Hence SUSY breaking seems non-generic and the known theories that have been studied so far all seem very complicated.

There is a way out. Unlike ordinary quantum field theory, SUSY theory often contains a large continuous manifold of vacua: "the moduli space", which offers an opportunity for rich vacuum structure. The seminal 2006 paper by K. Intriligator, N. Seiberg and D. Shih [2] (and related materials[3] [4]) proposed considering dynamical SUSY breaking in a meta-stable vacuum. The structure of vacua is crucial in determining the physical properties of most theoretical models. For instance, the vacuum structure determines whether the apparent symmetries of a model remain manifest or are spontaneously broken, and it is even more important to find out whether the vacuum structure of a model contains a unique and single ground state or more metastable states. This is the main spirit of ISS model, in which supersymmetry is broken by the metastable vacua, but at the same time there still exists a ground state to preserve the supersymmetry. The option that we can live in a metastable vacuum greatly loosens the restriction and opens up a whole lot of possible model building. After all, most of the restrictions we mentioned above apply to the vacuum of a SUSY theory. And for a model with SUSY breaking metastable state, the true vacuum can still be supersymmetric and hence escape the control of requirements imposed by SUSY. In Intriligator- Seiberg- Shih (ISS) model, the metastable vacuum is shown to be long-lived so that the nature we observe will be determined by its property instead of that of the true vacuum.

As an example, it was shown there is an inner connection between supersymmetry breaking and R-symmetry.[5] In general, there is broken supersymmetry if and only if there is an R-symmetry. For realistic model building, an unbroken R-symmetry is a serious problem. It forbids Majorana gaugino masses. However the connection applies only to the true vacuum. In the ISS model [2], SUSY is broken in a metastable state but the true SUSY vacuum ensures a broken R symmetry (an approximate R-symmetry, to be precise).

The ISS proposal sparks a renewed interest in metastable SUSY breaking. It is suggested that this phenomenon is quite generic in SUSY gauge theories. Despite its ubiquity, studying these metastable states in SUSY breaking scenarios is not easy. First of all, most models involves non-perturbative effects. Though the analysis of these non-perturbative dynamics has made huge progress recently, these tools applies mainly on the ground state. On the other hand, far fewer tools exists for calculating the properties of metastable states. Secondly, in most of the models, the distances between ground states and metastable states are large. This renders the low energy effective theory applicable to one unapplicable to the other due to the possible strong dynamics. Hence the calculation of a global property such as decay rate is either unreliable or too difficult to perform.

Our study in this thesis will concentrate on the possibility of metastable SUSY breaking at tree level. It is reasonable to speculate that if the model is complicated just enough, the SUSY scalar potential consisting of F-term and D-term will exhibit rich shapes and structures, such that metastable states will arise in addition to the true ground state. There are two scenarios in the literature that perturbatively realize a metastable state. In both cases, the vacuum structure can be determined perturbatively. This is a huge advantage over strongly interacting SUSY breaking models. The tunneling between the metastable state and the real vacuum can be calculated. We will study some physical implications of these models.

We first start with the modified O'Raiferataigh model. This model contains a field content and a superpotential that does not allow a zero F-term for the ground state. Hence a non-zero F-term breaks SUSY. The vacuum or vacua actually consists of a SUSY breaking pseudo-moduli space. In [3], it was shown that we can realize metastable state in this model. What is required is to add an small interaction term (a mass term for example) that would enable the F-term to vanish and generate a SUSY vacuum. On the other hand, if the additional interaction is small, we would expect the original SUSY breaking moduli vacuum space to stay the way it is but now becomes a metastable SUSY breaking state. Here we study the general features of this scenario and extend the superpotential to a more general form. We find very unusual moduli space structures in some choices.

The above modified O'Raiferataigh model of metastable SUSY breaking usually contains a small parameters that controls the location of SUSY vacuum. There is another possibility that doesn't require small parameters. In [6], K.R. Dienes and B. Thomas (2008) tried another approach. This is more or less a generalized hybrid of Wess-Zumino model [7] and Fayet-Illiopolous model [8]. Here multiple U(1) gauge interactions (and hence Fayet-Illiopolous terms (Fi terms)) are introduced and both D-term and F-term are present. Hence the tension among them produces a very rich vacuum structure, including several unstable states, a SUSY vacuum and a metastable SUSY breaking state. This model is complicated, requiring five chiral fields and two U(1) gauge groups. Of course, it's precisely this complication that enables a potential twisted enough to afford a rich vacuum structure. But we do want to know how we can simplify the model without ruining the metastable states. The model contains several parameters and features that can be adjusted. Hence we generalize by simplifying the model and demonstrate how the crucial features could be preserved. It turns out we can get away with only one U(1) gauge interaction. In some cases, we can even reduce the number of chiral superfields to three and further remove the mass term from the superpotential. The vacuum structure simply arise from the interplay between the two cubic terms in the superpotential and the U(1) FI term.

The outline of the thesis is as follows. In the next section, we first review the

O'Raifeartaigh model which breaks supersymmetry by non-zero F-term and how the metastable SUSY breaking could be realized here. Then in section 3, we extend the generic functions which plays an important role in O'Raifeartaigh model. In section 4, we consider the work of K.R. Dienes and B. Thomas[6] and vary the charges of U(1) gauge groups in their models in order to construct the metastable vacua to break SUSY. Finally, in section 5, we build our own model based on previous frameworks. Conclusions are given in section 6.

2. O'Raifeartaigh model and metastable SUSY breaking

Dating back to 1974, the O'Raifeartaigh model is a classic realization of breaking SUSY with non-zero F-term. Recently there has been a revival of interest in SUSY breaking using renormalizable perturbative models such as generalizations of the O'Raifeartaigh model. This is motivated in part because of the realization that O'Raifeartaigh type models will arise in the low energy effective theory of simple SUSY gauge theory. In the old days, its use in realistic model building has been hindered by an unbroken R symmetry that appears after SUSY is broken [5]. However, the possibility of metastable SUSY breaking provides a way to circumvent the problem.

In this section, we consider the supersymmetric theory with chiral superfields Φ^a. The supersymmetric Lagrangian is controlled by two functions of the superfields: The Kähler potential $K(\Phi,\overline{\Phi})$ and the superpotential $W(\Phi)$. The Lagrangian can be written as

$$\mathcal{L} = g_{a\bar{a}}(\partial_\mu \Phi^a \partial^\mu \overline{\Phi}^{\bar{a}} + i\bar{\psi}^a \bar{\sigma}^\mu \partial_\mu \psi_a) - \frac{1}{2}(\partial_{ab} W \psi^a \psi^b + \partial_{\bar{a}\bar{b}} W \bar{\psi}^a \bar{\psi}^b) - V(\Phi, \overline{\Phi}) \quad (2.1)$$

with the metric function $g_{a\bar{a}}$ given by the derivative of the Kähler potential with respect to superfields and after differentiation the superfields are set equal to their corresponding scalar fields:

$$g_{a\bar{a}} = \partial_a \partial_{\bar{a}} K. \quad (2.2)$$

The all important scalar potential is given as the square of the superfield component $\mathcal{F}$

$$V = \mathcal{F}^a \overline{\mathcal{F}_{\bar{a}}} = g^{a\bar{a}} \partial_a W \partial_{\bar{a}} \overline{W} \quad (2.3)$$

As shown here, $\mathcal{F}$ is only an auxiliary field and according to the equation of motion and it is equal to the derivative of the superpotential with respect to superfields.

According to the SUSY algebra, the energy can be written as a positive definite sum of products of spinor generators:

$$Q_1\bar{Q}_1 + \bar{Q}_1 Q_1 + Q_2 \bar{Q}_2 + \bar{Q}_2 Q_2 = 4P^0 \quad (2.4)$$

If the vacuum is supersymmetric, i.e.

$$Q|0\rangle = 0, \quad (2.5)$$

the energy of the vacuum is equal to zero. Ground state energy becomes the order parameter of spontaneous SUSY breaking. For a SUSY vacuum to exist, the scalar potential V must vanish. Since V only contains the sum of squares of $\mathcal{F}$, all the $\mathcal{F}$ terms must vanish for SUSY vacuum[1]. Therefore, we can solve

$$\overline{\mathcal{F}_{\bar{a}}} = \partial_a W(\Phi) = 0 \qquad \forall a \tag{2.6}$$

to search for SUSY vacuum or vacua. This result implies a significant difference from ordinary quantum field theory. While vacuum structure is relatively simple in the latter, in the former the scalar potential may be complicated enough to admit more than one supersymmetric vacuum. For example, if W is a function of n degree of one superfield Φ, there could be n-1 solution to (2.6) and hence n-1 supersymmetric vacua. On the other hand, it is also possible that the superpotential is so chosen that there are no such points to satisfy (2.6) at all. Then the supersymmetric ground states do not exist and SUSY is spontaneously broken.

2.1 *O'Raifeartaigh model*

O'Raifeartaigh model is the simplest example of spontaneous supersymmetry breaking by non-zero F-term. We can start the discussion in general form. Assume that there are a set of r chiral superfields X_i and another set of s chiral superfields ϕ_j ,with $r > s$. We assume a special form for the superpotential

$$W = \sum_{i=1}^{r} X_i g_i(\phi_j) \tag{2.7}$$

There is a $U_R(1)$ symmetry by the assignment of $R(X_i) = 2$ and $R(\phi_j) = 0$. Note that chiral superfields X_i only communicate with each other through ϕ_j. For generic functions g_i, it is impossible for all $-\overline{\mathcal{F}}_{X_i} = g_i(\phi_j)$ to vanish simultaneously since there are more (r) equations than (s) unknowns. Hence supersymmetry is broken.

The vacuum structure of this model is very interesting. It is determined at the tree-level by the scalar potential:

$$V_{tree} = \sum_{i=1}^{r} |g_i(\phi_j)|^2 + \sum_{j=1}^{s} |\sum_{i=1}^{r} X_i \frac{\partial}{\partial \phi_j} g_i(\phi_j)|^2 \tag{2.8}$$

Supersymmetric theories have the unique property that there often for a continuous manifold of vacua known as "moduli space of vacua". (Nevertheless, if we take the quantum effect into account, then the non-supersymmetric degeneracy of vacua is

[1] Here we do not include gauge fields. In later section we will take gauge theory into account, and then the scalar potential V will involve another auxiliary field $\mathcal{D}$, namely $V = \mathcal{F}^a \overline{\mathcal{F}_a} + \frac{1}{2}\mathcal{D}^a \overline{\mathcal{D}}_a$.

lifted. As a result, we prefer to call this space as pseudo-moduli space of vacua.) From the extremum conditions of (2.8):

$$\frac{\partial V_{tree}}{\partial X_i} = 0, \tag{2.9}$$

it could be shown the VEV's of the r fields X_i are governed only by s conditions:

$$\sum_{i=1}^{r} X_i \frac{\partial}{\partial \phi_j} g_i(\phi_j) = 0 \qquad \forall j = 1 \ldots s, \tag{2.10}$$

Therefore for generic functions $g_i(\phi_j)$, the VEV's of X_i still has a dimension of $r-s$ to vary and there are a whole space of vacua. This pseudo-moduli space of vacua has complex dimension equal to $r-s$. The X_i equations of motion are automatically satisfied on this pseudo-moduli space. The ϕ_j are determined by their equations of motion,

$$\sum_{i=1^s} \overline{g_i(\phi_j)} \frac{\partial}{\partial \phi_j} g_i(\phi_j) = 0 \qquad \forall j = 1 \ldots s \tag{2.11}$$

which is generally satisfied for a discrete set of values $\phi_j = \phi_j^{(n)}$, some of which are local minima of the potential (others are saddle points).

To summarize, the tree-level potential generally has a pseudo-moduli space of supersymmetry breaking vacua, labeled by the $r-s$ complex dimensional space of expectation values of the X_i, subject to (2.10), along with some discrete choices $\phi_j^{(n)}$ of the solutions of (2.11). The pseudo-moduli are lifted at one loop, by the Coleman-Weinberg effective potential,

$$V_{eff} = \frac{1}{64\pi^2} STr \mathcal{M}^4 log \frac{\mathcal{M}^2}{M^2_{cutoff}} \tag{2.12}$$

where $\mathcal{M}$ is the classical, pseudo-moduli-dependent mass matrix, and STr denotes supertrace. The resulting vacua, in all these examples, always have $\langle X_i \rangle = 0$ as the minimum of the one-loop potential. The $U(1)_R$ symmetry is thus not spontaneously (nor explicitly) broken in these examples.

In the original form[9], the O'Raifeartaigh model has three chiral superfields, ϕ, X_1 and X_2 with canonical Kähler potential $K = K_{can} = \phi\overline{\phi} + X_1\overline{X}_1 + X_2\overline{X}_2$ and the superpotential is

$$W_{OR_1} = \frac{1}{2} h\phi^2 X_1 + f X_1 + m\phi X_2 \tag{2.13}$$

For convenience, we call the original O'Raifeartaigh model as OR_1. We can understand that $g_1(\phi) = \frac{1}{2}h\phi^2 + f$ and $g_2(\phi) = m\phi$. By (2.8), its scalar potential at tree-level is

$$V_{OR_1} = |\mathcal{F}_\phi|^2 + |\mathcal{F}_{X_1}|^2 + |\mathcal{F}_{X_2}|^2 = |h\phi X_1 + mX_2|^2 + |\frac{1}{2}h\phi^2 + f|^2 + |m\phi|^2 \tag{2.14}$$

There is an $r - s = 1$ complex dimensional pseudo-moduli space of vacua, with X_1 and X_2 constrained by the single condition (2.10) which here gives $hX_1\phi + mX_2 = 0$. The ϕ equation of motion (2.11) leads to two cases, depending on the value of

$$y \equiv \left|\frac{hf}{m^2}\right| \tag{2.15}$$

Consider first the case of $y < 1$. Then the potential is minimized by the $\mathcal{F}_{X_2} = 0$ at $\langle\phi\rangle = \langle X_2\rangle = 0$ and arbitrary $\langle X_1\rangle$, with value

$$V_{min} = |\mathcal{F}_{X_1}|^2 = |f|^2 \neq 0. \tag{2.16}$$

These vacua breaks supersymmetry because $V_{min} \neq 0$.

The spectrum can be determined easily. The fermion ψ_{X_1} is the exactly massless Goldstino. The scalar component of X_1 is a classical pseudo-moduli. The classical mass spectrum of the ϕ and X_2 fields can be easily computed. For the two, two-component fermions, the eigenvalues are

$$m_{1/2}^2 = \frac{1}{4}(|hX_1| \pm \sqrt{|hX_1|^2 + 4|m|^2})^2 \tag{2.17}$$

and for the four real scalars the mass eigenvalues are

$$m_0^2 = |m|^2 + \frac{1}{2}\eta|hf| + \frac{1}{2}|hX_1|^2 \pm \frac{1}{2}\sqrt{|hf|^2 + 2\eta|hf||hX_1|^2 + 4|m|^2|hX_1|^2 + |hX_1|^4} \tag{2.18}$$

where $\eta = \pm 1$. These masses satisfy a sum rule of tree level breaking

$$Tr[\mathcal{M}_0^2] = 2Tr[\mathcal{M}_{1/2}^2] \tag{2.19}$$

where $\mathcal{M}$ is the mass matrix. The classical pseudo-moduli space degeneracy is lifted at one-loop by the Coleman-Weinberg potential(2.12). The formula is complicated, but near $X_1 = 0$, the potential is approximately given by

$$V_{eff}^{(1)}(X_1) = V_0 + m_{X_1}^2|X_1|^2 + \mathcal{O}(|X_1|^4) \tag{2.20}$$

where V_0 is a constant and

$$\begin{aligned} m_{X_1}^2 &= \tfrac{1}{32\pi^2}\,|h^2m^2|\,f_1(y) \\ f_1(y) &\equiv y^{-1}\left((1+y)^2\log(1+y) - (1-y)^2\log(1-y) - 2y\right) \end{aligned} \tag{2.21}$$

Since $f_1(y)$ is positive for all $y < 1$, the potential (2.20) has an $U(1)_R$-preserving minimum at $X_1 = 0$.

On the other hand, in the case of $y > 1$, there are two disjoint pseudo-moduli spaces given by

$$\langle X_2 \rangle = -\frac{h}{m}\phi X_1, \qquad \langle \phi \rangle = \pm i\sqrt{\frac{2f}{h}(1-y^{-1})} \tag{2.22}$$

and X_1 is arbitrary.[2] The Coleman-Weinberg potential again takes the form (2.12), now with

$$\begin{aligned} m_{X_1}^2 &= \tfrac{1}{8\pi^2}\left|h^2 m^2\right| f_2(y) \\ f_2(y) &\equiv y^2 \log y - (y-1)^2 \log(y-1) - (y-1/2)\left(2\log\left(y-1/2\right)+1\right) \end{aligned} \tag{2.23}$$

Since $f_2(y) > 0$ for all $y > 1$, the one-loop effective potential is minimized at $X_1 = 0$, and the $U(1)_R$ remains unbroken.

2.2 *Metastable supersymmetry breaking in a modified O'Raifeartaigh model*

According to above discussion, we argue that (2.7) breaks SUSY for generic functions $g_i(\phi_j)$. Now we consider a simple example of superpotential which contains non-generic $g_1(\phi)$ and $g_2(\phi)$. In this example, we can preserve supersymmetry. Nevertheless, supersymmetry could be broken by a metastable vacuum.

The non-generic functions are chosen as follow

$$g_1(\phi) = h\phi(\phi - m_1), \qquad g_2(\phi) = m_2(\phi - m_1) \tag{2.24}$$

therefore according to (2.7) the superpotential is

$$W = g_1 X_1 + g_2 X_2 = h\phi(\phi - m_1)X_1 + m_2(\phi - m_1)X_2 \tag{2.25}$$

and from (2.8) we can get the scalar potential at tree level as below,

$$V = \left|h\phi^2 - m_1 h\phi\right|^2 + \left|m_2\phi - m_1 m_2\right|^2 + \left|2h\phi X_1 - m_1 h X1 + m_2 X_2\right|^2 \tag{2.26}$$

As the same as the previous discussion of O'Raifeartaigh model, the equations of motion for X_1, X_2 and ϕ are obtained by (2.10) and (2.11)

$$X_1(2h\phi - m_1 h) + X_2 m_2 = 0 \tag{2.27}$$

$$(\phi - m_1)(2h^2\phi^2 - m_1 h^2\phi + m_2^2) = 0 \tag{2.28}$$

[2] There can be another choice of $\langle \phi \rangle = \pm\sqrt{-\frac{2f}{h}(1+y^{-1})}$ in the condition of $hf < 0$, $h < 0$, and $-\sqrt{-hf} < m < \sqrt{-hf}$ (or $h > 0$, and $-\sqrt{-hf} < m < \sqrt{-hf}$).

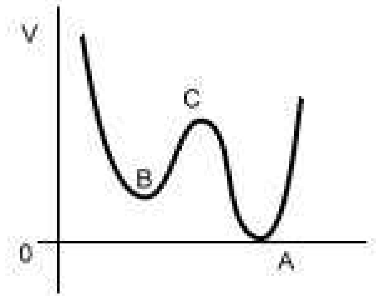

Figure 1: *The scalar potential of non-generic superpotential has the metastable vacua (B), unstable vacua (C) and supersymmetric vacua (A) as well.*

From (2.28) we can see that the chiral field $\langle\phi\rangle$ has three different values which are m_1 and $\frac{1}{4}(m_1 \pm \frac{1}{h}\sqrt{m_1 h^2 - 8m_2^2})$. Supersymmetry can be preserved at

$$\langle\phi\rangle_{susy} = m_1, \qquad \langle X_2\rangle_{susy} = -\frac{hm_1}{m_2}\langle X_1\rangle_{susy} \tag{2.29}$$

with arbitrary $\langle X_1\rangle_{susy}$ which is obtained from (2.27). On the other hand, there is a metastable pseudo-moduli vacua when we choose $\langle\phi\rangle = \frac{1}{4}(m_1 \pm \frac{1}{h}\sqrt{m_1 h^2 - 8m_2^2})$ and $\langle X_2\rangle = -\frac{hm_1}{m_2}\langle X_1\rangle$ with still arbitrary $\langle X_1\rangle$ with the condition of $\left|\frac{hm_1}{m_2}\right|^2 > 8$. These metastable vacua breaks supersymmetry because they not only satisfy the minima of the potential but also have the greater values of potential than supersymmetric potential. The relation of metastable vacua and supersymmetric vacua can be illustrated in Figure 1

So far, we have the brief understanding of supersymmetry breaking by metastable vacua. Now we turn to modify O'Raifeartaigh model by adding a small correction into the original superpotential,

$$W = \frac{1}{2}hX_1\phi^2 + fX_1 + m\phi X_2 + \frac{1}{2}\epsilon m X_2^2 \tag{2.30}$$

with $|\epsilon| \ll 1$. This added term is effective because it not only breaks $U(1)_R$ but also breaks SUSY by metastable vacua. The scalar potential is modified to

$$V = |h\phi X_1 + mX_2|^2 + |\frac{1}{2}h\phi^2 + f|^2 + |m\phi + \epsilon X_2|^2 \tag{2.31}$$

As the result of added term in (2.30), SUSY can be preserved at two supersymmetric vacua

$$\langle\phi\rangle_{susy} = \pm\sqrt{\frac{-2f}{h}}, \qquad \langle X_1\rangle_{susy} = \frac{m}{h\epsilon}, \qquad \langle X_2\rangle_{susy} = \mp\frac{1}{\epsilon}\sqrt{\frac{-2f}{h}} \tag{2.32}$$

and the previous minimum (2.16) breaks SUSY at $\langle\phi\rangle = \langle X_2\rangle = 0$ and arbitrary $\langle X_1\rangle$. However, these vacua play the metastable vacua to break supersymmetry.

Precisely, we still need to confirm the scalar masses (2.18) still remain non-tachyonic situation. Therefore there is a constraint of

$$\left|X_1 - \frac{m}{h\epsilon}\right|^2 > \left(\frac{1}{|\epsilon|^2} + 1\right)\left|\frac{f}{h}\right| \tag{2.33}$$

In the range of (2.33), the pseudo-moduli space is locally stable.

2.3 *Supersymmetry breaking and R-symmetry*

Now, we consider the relation between SUSY breaking and R-symmetry. As we mentioned in the introduction section, there is supersymmetry breaking if and only if there must be R-symmetry in the theory. In this section, we discuss the general conclusion of R-symmetry and SUSY breaking not only in O'Raifeartaigh but also in metastable cases. Take the generic condition of supersymmetry breaking into account

$$\partial_a W(\Phi) = 0 \qquad \forall a = 1 \ldots k \tag{2.34}$$

we can not solve all of the equations. However, if W is generic, there are k equations for the k Φ^a fields. Therefore we can solve the equations generally. Till now there have not involved R-symmetry in the superpotential. If we put the a global non-R U(1) symmetry into the superpotential, then the condition (2.34) is modified as below

$$W = W(t^a = \Phi^a \Phi_1^{-q_a/q_1}) \qquad a = 2 \ldots k \tag{2.35}$$

q_a is the U(1) charge of Φ^a. There are k-1 equations for k-1 independent unknowns therefore (2.34) can be solved. Moreover, if there is an R-symmetry in the superpotential, then

$$W = Tf(t^a = \Phi^a \Phi_1^{-r_a/r_1}) \qquad T = \Phi_1^{2/r_1} \tag{2.36}$$

where r_a is the R-charge of Φ^a. The equations can not be solved because there are k equations for k-1 independent unknowns. As the result, SUSY is broken.

The O'Raifeartaigh type models of (2.7) have an R-symmetry and broken supersymmetry because of generic $g_1(\phi)$ and $g_2(\phi)$. However, the superpotential of (2.25) have an R-symmetry and preserve supersymmetry because the superpotential is not generic.

Consider the modified O'Raifeartaigh model and the relation between R-symmetry and SUSY breaking by metastable vacua. As the added deformation term in (2.30) which breaks R-symmetry and restores supersymmetry indeed. But for small ϵ there is an approximate R-symmetry which is related to supersymmetry breaking in the metastable state.

To summarize, generically there is broken supersymmetry if and only if there is an R-symmetry in superpotential. And there is broken supersymmetry by metastable vacua if and only if there is an approximate R-symmetry in superpotential. For realistic models of SUSY breaking, it is a must to break R-symmetry to get gaugino masses.

3. Generalizing the generic functions in O'Raifeartaigh model

In this chapter we investigate the vacuum structure of the general O'Raifeartaigh model for some functions $g_i(\phi)$ with higher power. Different generic functions lead to different superpotential correspondingly. Here we didn't change the form of superpotential, but concentrate on modifying the order of generic functions. For convention, we call the original O'Raifeartaigh model as OR_1 and the following models as OR_2, OR_3 etc.

3.1 *The OR_2 model*

Consider the generic functions $g_1(\phi) = \frac{1}{2}h_1\phi^2 + f$ and $g_2(\phi) = \frac{1}{2}h_2\phi^2$[3]. By (2.7), we get the superpotential as below

$$W_{OR_2} = \frac{1}{2}h_1\phi^2 X_1 + fX_1 + \frac{1}{2}h_2\phi^2 X_2 \tag{3.1}$$

and the scalar potential is

$$V_{OR_2} = |h_1\phi X_1 + h_2\phi X_2|^2 + |\frac{1}{2}h_1\phi^2 + f|^2 + |\frac{1}{2}h_2\phi^2|^2 \tag{3.2}$$

This OR_2 breaks SUSY as OR_1 does. Nevertheless, it has a different vacua structure. By solving the (2.10) and (2.11)

$$\phi(h_1X_1 + h_2\phi X_2) = 0 \tag{3.3}$$

and

$$\phi^*(\frac{1}{2}h_1^2|\phi|^2 + \frac{1}{2}h_2^2|\phi|^2) + h_1 f\phi = 0 \tag{3.4}$$

There are two different solutions of (3.3). One is $\langle\phi\rangle = 0$ and the other is $X_2 = -\frac{h_1}{h_2}X_1$. If $\langle\phi\rangle \neq 0$, it is given by the second equation $\langle\phi\rangle = \pm i\sqrt{\frac{2h_1 f}{h_1^2+h_2^2}}$ (or $\pm\sqrt{\frac{-2h_1 f}{h_1^2+h_2^2}}$). It could be shown that the scalar potential is smaller (but non-zero) at $\langle\phi\rangle = \pm i\sqrt{\frac{2h_1 f}{h_1^2+h_2^2}}$ and $X_2 = -\frac{h_1}{h_2}X_1$ and this is the true vacuum.

As expected, the vacuum is a one dimensional SUSY breaking moduli space. But it can be shown that the spectrum is supersymmetric and the one-loop effective potential correction vanishes. This is because there is a unitary transformation of chiral fields X_1 and X_2.

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = U \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \tag{3.5}$$

where U is an U(2) unitary matrix. In terms of new fields, the superpotential is

$$W_{OR_2} \rightarrow \frac{1}{2}hY_1(f_1+\phi^2)+f_2Y_2 \tag{3.6}$$

Therefore the theory is decoupled into two independent sectors. The supersymmetry is preserved in the first sector with a supersymmetric vacuum $\phi = \pm i\sqrt{f_1}$ and $Y_1 = 0$. However, the second sector of the decoupled Y_2 field breaks supersymmetry instead. Actually this theory is trivial because it is a free field theory without any interaction term.

3.2 *The OR_3 model and the OR_4 model*

Now we raise the power of the function g's and assume: $g_1(\phi) = \frac{1}{3}h_1\phi^3 + f$ and $g_2(\phi) = \frac{1}{2}h_2\phi^2$. We can write down the superpotential and the scalar potential as usual,

$$W_{OR_3} = \frac{1}{3}h_1\phi^3X_1 + fX_1 + \frac{1}{2}h_2\phi^2X_2 \tag{3.7}$$

and

$$V_{OR_3} = |h_1\phi X_1 + h_2\phi X_2|^2 + |\frac{1}{3}h_1\phi^3 + f|^2 + |\frac{1}{2}h_2\phi^2|^2 \tag{3.8}$$

This OR_3 model breaks SUSY and has quite similar vacua structure as OR_2 model which can been obtained by solving (2.10) and (2.11).

$$\phi(h_1\phi X_1 + h_2X_2) = 0 \tag{3.9}$$

and

$$\frac{1}{3}h_1^2(|\phi|^2)^2\phi^* + h_1f\phi^2 + \frac{1}{2}h_2^2|\phi|^2\phi^* = 0 \tag{3.10}$$

There are two solutions characterized by the different expectation value of $\phi: \langle\phi\rangle = 0$ or $\langle\phi\rangle$ is the real roots[3] of $\frac{1}{3}h_1^2\phi^3 + \frac{1}{2}h_2^2\phi + h_1f = 0$. Which one has the smaller potential will depend on the parameters. In the case of $\langle\phi\rangle = 0$, there are two arbitrary pseudo-moduli space field X_1 and X_2. . On the other hand, if $\langle\phi\rangle \neq 0$, there is only one arbitrary field as determined by $X_2 = -\frac{h_1}{h_2}\phi X_1$.

As an example, let's study the case $\langle\phi\rangle = 0$. We can calculate the masses of bosons and fermions at this vacuum:

$$m_0^2 = h_2^2X_2^2, \qquad h_2^2X_2^2, \qquad 0, \qquad 0, \qquad 0, \qquad 0 \tag{3.11}$$

and

$$m_{1/2}^2 = h_2^2X_2^2, \qquad 0, \qquad 0 \tag{3.12}$$

[3]The roots can be imaginary. But for simplicity, we always discuss the real solutions.

It is a surprise that even though supersymmetry is broken, the spectrum of the massive fields is supersymmetric indeed. As in the previous OR_2 model, the superpotential of OR_3 model can be separated into two decoupled sectors by an U(2) unitary matrix.

We further try to change the power of $g_2(\phi)$. Now we assume $g_1(\phi) = \frac{1}{3}h_1\phi^3 + f$ and $g_2(\phi) = h_2\phi$, and call this model OR_4. The superpotential and the scalar potential can be written as,

$$W_{OR_4} = \frac{1}{3}h_1\phi^3 X_1 + fX_1 + h_2\phi X_2 \tag{3.13}$$

and

$$V_{OR_4} = |h_1\phi^2 X_1 + h_2 X_2|^2 + |\frac{1}{3}h_1\phi^3 + f|^2 + |h_2\phi|^2 \tag{3.14}$$

The OR_4 model also breaks SUSY as we can see from (3.14). In addition, we can discuss the constrain equations of chiral fields and pseudo-moduli space by considering (2.10) and (2.11)

$$h_1\phi^2 X_1 + h_2 X_2 = 0 \tag{3.15}$$

and

$$\frac{1}{3}h_1^2(|\phi|^2)^2\phi^* + h_1 f\phi^2 + h_2^2\phi^* = 0 \tag{3.16}$$

By solving (3.16), we can obtain different solutions of ϕ which includes $\langle\phi\rangle = 0$ or $\langle\phi\rangle$ is the real solutions of $\frac{1}{3}h_1^2\phi^4 + h_1 f\phi^2 + h_2^2\phi = 0$. As we turn our attention to (3.15), then we have noticed the ϕ^2 dependent solution which is $X_2 = -\frac{h_1}{h_2}\phi^2 X_1$ with an arbitrary X_1 chiral field. If $\langle\phi\rangle = 0$, then the vacua structure is the same as one sort of vacua structure of OR_1 model which is $\langle\phi\rangle = \langle X_2\rangle = 0$ with arbitrary X_1.

As we choose $\langle\phi\rangle = 0$, we can have the fermions and bosons spectrum as below

$$m_0^2 = h_2^2, \qquad h_2^2, \qquad h_2^2, \qquad h_2^2, \qquad 0, \qquad 0 \tag{3.17}$$

and

$$m_{1/2}^2 = h_2^2, \qquad h_2^2, \qquad 0 \tag{3.18}$$

which are the same as the OR_3 model, these massive spectrums are still supersymmetric. However both fermions and bosons masses are constants h_2^2 which are different from the OR_3 model whose masses are X_2-dependent.

3.3 *The OR_5 model*

The previous models all involve the same choice of generic functions which the order of $g_1(\phi)$ is greater than $g_2(\phi)$. In this subsection, we consider the opposite situation in which $g_1(\phi) = \frac{1}{2}h_1\phi^2 + f$ and $g_2(\phi) = \frac{1}{3}h_2\phi^3$. Therefore the superpotential and scalar potential of OR_5 model can be written down for these generic functions.

$$W_{OR_5} = \frac{1}{2}\phi^2 X_1 + fX_1 + \frac{1}{3}h_2\phi^3 X_2 \tag{3.19}$$

and

$$V_{OR_5} = |h_1\phi X_1 + h_2\phi^2 X_2|^2 + |\frac{1}{2}h_1\phi^2 + f|^2 + |\frac{1}{3}h_2\phi^3|^2 \tag{3.20}$$

As the previous OR-type models, the OR_5 model also breaks SUSY from (3.20). We also can calculate the equation of chiral fields and pseudo-moduli space

$$\phi(h_1X_1 + h_2\phi X_2) = 0 \tag{3.21}$$

and

$$\frac{1}{2}h_1^2|\phi|^2\phi^* + h_1f\phi + \frac{1}{3}h_2^2(|\phi|^2)^2\phi^* = 0 \tag{3.22}$$

As we can see from (3.21), we can obtain the solution $\langle\phi\rangle = 0$ and $X_2 = -\frac{h_1}{h_2}\frac{X_1}{\phi}$. There is a different situation of the X_2 compared to the previous models. The X_2 is ϕ^{-1}-dependent. Here we can give a small summary about the ϕ-dependence of X_2 which X_2 has the ϕ order of $\mathcal{O}(g_1(\phi) - g_2(\phi))$. Moreover, if $\langle\phi\rangle = 0$, then X_1 and X_2 are arbitrary from solving (3.21) which is the same as OR_2 and OR_3. On the other hand, if $\langle\phi\rangle \neq 0$, then $\langle\phi\rangle = (\frac{1}{h_2^2}(-\frac{3}{4}h_1^2 \pm \sqrt{\frac{9}{16}h_1^2 - 3h_2^2h_1f}))^{\frac{1}{2}}$

Also, the bosons and fermions masses are obtained under the condition of $\langle\phi\rangle = 0$ as below

$$m_0^2 = h_1^2X_1^2 \pm h_1f, \qquad 0, \qquad 0, \qquad 0, \qquad 0 \tag{3.23}$$

and

$$m_{\frac{1}{2}}^2 = h_1^2X_1^2, \qquad 0, \qquad 0 \tag{3.24}$$

The form of massive fields spectrum is quite similar to the OR_2 model but without X_2-dependent in the OR_5 model.

3.4 *Summary*

According to the previous discussion, we can observe that the vacua structure and the fermions and bosons masses have the relation with the order of generic functions $g_1(\phi)$ and $g_2(\phi)$.

First, we have mentioned that the X_2 has the ϕ-dependence relative to the difference of order of two generic functions $g_1(\phi)$ and $g_2(\phi)$. Moreover, when both the order of $g_1(\phi)$ and $g_2(\phi)$ are greater than 2, there are two arbitrary pseudo-moduli X_1 and X_2. The reason can be traced back to (2.10), as we can notice that the partial derivative reduces one order from generic functions which implies that there will be co-factor ϕ which is equal to zero if both generic functions have order 2 or larger than 2. As a result,the remaining part of the constraint of (2.10) can be arbitrary. For example, in the OR_2, OR_3 and OR_5 models, the equation of motion of pseudo-moduli (3.3), (3.9) and (3.21) all satisfy both generic functions whose order are higher than 2 and all have two arbitrary pseudo-moduli space X_1 and X_2.

Second, the spectrum of boson and fermion fields are obtained from (2.3) and (2.1). Because the mass terms are obtained by the second order partial derivative, the order of generic functions can not be higher than 2 otherwise there will be no contribution in mass terms under the condition of $\langle\phi\rangle = 0$. Apparently in the OR_3 and OR_4 models, the fermion and boson masses are supersymmetric due to the higher order of $g_1(\phi)$ and it leads terms that are still ϕ-dependent and these terms are zero as $\langle\phi\rangle = 0$ and the remaining terms are either X_2-dependent or constants which are determined by the order of $g_2(\phi)$. As we can compare the OR_2-OR_5 and OR_3-OR_4, the generic function $g_2(\phi)$ plays the role in determining the X_2-dependence in the mass spectrum.

Finally, all the OR-type models break supersymmetry and preserve R-symmetry which satisfy the relation between supersymmetry and R-symmetry.

4. Classical metastablility and nontrivial vacuum structure

The above modified O'Raiferataigh model of metastable SUSY breaking usually contains a small parameter that controls the location of SUSY vacuum. In [6], K.R. Dienes and B. Thomas (2008) proposed another approach that doesn't require small parameters. They introduced U(1) gauge interactions and nonzero Fayet-Illiopolous terms. Hence both D terms and F terms are present in the scalar potential and the tension among them produces a very rich vacuum structure. This complexity of the scalar potential provides a fertile ground for metastable states.

4.1 *The model of Dienes and Thomas*

The Dienes' model is more or less a generalized hybrid of Wess-Zumino model and Fayet-Illiopolous model. It involves two U(1) gauge groups denoted as $U_a(1)$ and $U_b(1)$ with correspondingly gauge coupling constant g_a and g_b and Fayet-Iliopoulos terms ξ_a and ξ_b. There are five chiral superfields with the following charges:

Field	$U(1)_a$	$U(1)_b$	$U(1)_R$
Φ_1	-1	0	2/3
Φ_2	$+1$	-1	2/3
Φ_3	0	$+1$	2/3
Φ_4	$+1$	$+1$	1
Φ_5	-1	-1	1

Table 1: The field content and charge assignments for Dienes' model under consideration.

They introduced a superpotential as below:

$$W = \lambda\Phi_1\Phi_2\Phi_3 + m\Phi_4\Phi_5 \tag{4.1}$$

This is the most general superpotential we can write down that are consistent with all the symmetries and renormaliality The two U(1) symmetries are very restricted indeed.

The scalar potential in this model contains both D-terms and F-terms.

$$V = \overline{\mathcal{F}}_i\mathcal{F}_i + \frac{1}{2}g_i^2\mathcal{D}_i^2 \tag{4.2}$$

where

$$\mathcal{F}_i = -\frac{\partial W}{\partial \phi_i}, \qquad \mathcal{D}_i = \xi_i + \sum_j q_j^{(i)}|\phi_j|^2. \tag{4.3}$$

The scalar potential can be worked out as below

$$\begin{aligned} V &= \lambda^2(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2) + m^2(|\phi_4|^2 + |\phi_5|^2) \\ &+ \frac{1}{2}g_a^2(\xi_a + a_1|\phi_1|^2 + a_2|\phi_2|^2 + a_3|\phi_3|^2 + a_4|\phi_4|^2 + a_5|\phi_5|^2)^2 \\ &+ \frac{1}{2}g_b^2(\xi_b + b_1|\phi_1|^2 + b_2|\phi_2|^2 + b_3|\phi_3|^2 + b_4|\phi_4|^2 + b_5|\phi_5|^2)^2 \ . \end{aligned} \tag{4.4}$$

where a_i and b_i ($i = 1\ldots5$) are $U(1)$ gauge charges of each chiral superfield. As an illustration, we can consider the case where the gauge couplings are equal, $g_a = g_b = 1$ and the remaining constants are $(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)$. The general case will be discussed in the next subsection. The $U(1)$ and $U(1)_R$ gauge charges of these chiral fields are as shown in Table 1[4]

So the scalar potential becomes

$$\begin{aligned} V &= |\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_4|^2 + |\phi_5|^2 \\ &+ \frac{1}{2}(5 - |\phi_1|^2 + |\phi_2|^2 + |\phi_4|^2 - |\phi_5|^2)^2 \\ &+ \frac{1}{2}(-|\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 - |\phi_5|^2)^2 \ . \end{aligned} \tag{4.5}$$

The main task is to understand the vacua structure. We shall search the extrema first. The extrema satisfy the extrema condition of scalar potential (4.5).

$$\frac{\partial V}{\partial \phi_i} = 0 \qquad i = 1\ldots5 \tag{4.6}$$

To decide whether the extrema we found are stable, metastable or unstable, we have to check the mass matrix around the extremum. The extremum is stable if and only if mass matrix evaluated there contains all non-negative eigenvalues and the proper number of zeros, which corresponds to the Goldstone bosons to be absorbed. In this case, there should be two goldstone bosons because of two Abelian gauge groups. (Indeed, the additional zeros of eigenvalues of mass matrix indicate the presence of moduli spaces.) The mass matrix can be obtained by

$$\mathcal{M}^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial\phi_i\partial\phi_j} & \frac{\partial^2 V}{\partial\phi_i\partial\phi_j^*} \\ \frac{\partial^2 V}{\partial\phi_i^*\partial\phi_j} & \frac{\partial^2 V}{\partial\phi_i^*\partial\phi_j^*} \end{pmatrix} \tag{4.7}$$

Based on these conditions, we find there are one SUSY vacuum, one metastable SUSY breaking state and several other unstable extrema (of which I list only one, for

[4] There are the mixed anomalies to be concerned. According to [10], the mixed anomalies may be canceled by adding moduli fields S_n which does not play a role in breaking SUSY.

a complete lists see [6]). These states and their corresponding properties are listed in Table 2 [5]

Label	$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$	V	Stability	SUSY	R- symmetry
A	$(\sqrt{5}, 0, 0, 0, 0)$	0	Stable	Yes	Yes
B	$(0, 0, \sqrt{2}, 0, \sqrt{2})$	$\frac{9}{2}$	Metastable	No	No
C	$(\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{7}}{2}, 0, \sqrt{\frac{5}{2}})$	$\frac{45}{8}$	Unstable	No	No

Table 2: *The vacuum structure of Dienes' model.*

In this model, the scalar potential allows a SUSY ground state or vacuum. Note that its potential is equal to zero. In addition to that, it also generate a metastable SUSY breaking state. The two are separated by a large but calculable distance.

To illustrate the shape of the potential, we draw a 3D-shadow graph in Figure 2 and the contour graph in Figure 3. These diagram are drawn of the scalar potential on the two-dimensional plane within the three-dimensional (ϕ_1, ϕ_3, ϕ_5) field space which simultaneously contains points A, B and C. The values of (ϕ_1, ϕ_3) for this two dimensional plane are chosen to be shown as the (x, y) coordinates. ϕ_2 and ϕ_4 are set to zero. Moreover we choose the protectory points of supersymmetry and metastable vacua in field space to draw 2D graph of the scalar potential vs field space in Figure 4. In these diagrams, the relative positions and the respective values of scalar potential can be easily seen. The point B is indeed metastable, separated from the true vacuum A by the unstable extremum C.

[5]Here the value of ϕ_i $(i = 1 \dots 5)$ indicates the expectation value of each chiral superfield.

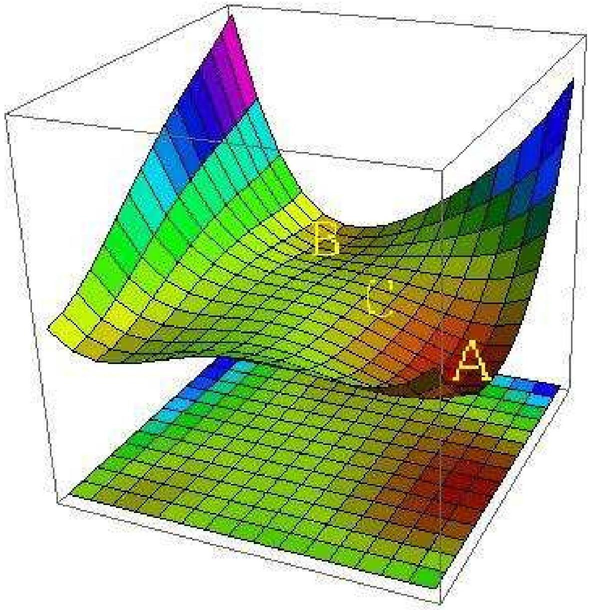

Figure 2: *The surface plot of the scalar potential V on the plane containing points A,B,C vs the VEV's of* (ϕ_1, ϕ_3).

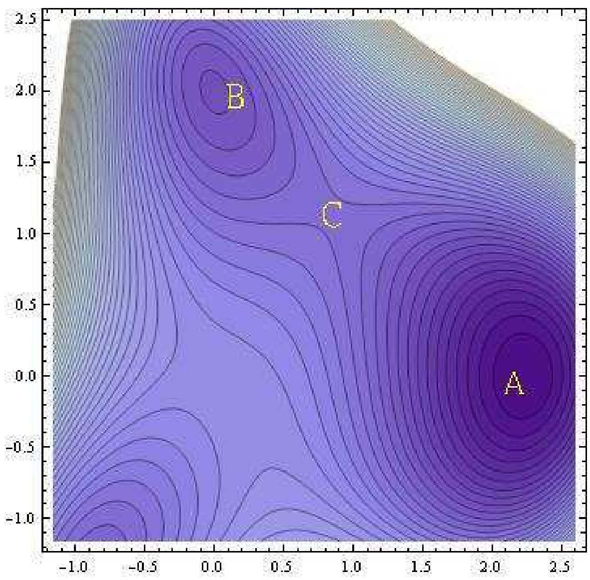

Figure 3: *The contour plot for V.*

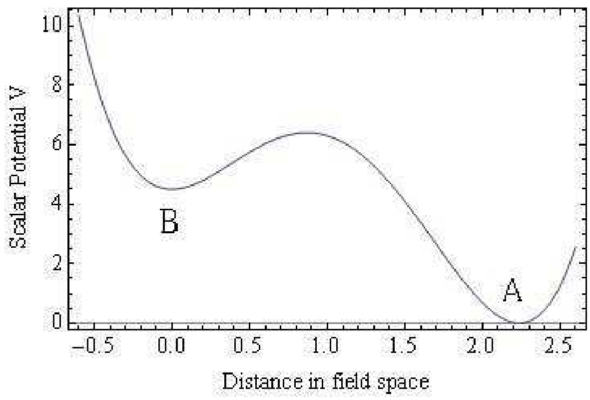

Figure 4: *The scalar potential is illustrated by different points which are on the line of SUSY and metastable vacua.*

4.2 *Exploring the parameter space*

We have demonstrated a model which contains both supersymmetry and metastable vacua simultaneously. This model a special case based on the particular parameters choice $(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)$ and $g_a = g_b = 1$. Now we will discuss the proper range of parameters and the gauge charges in which the vacua structure contains metastable vacua.

In Dienes' paper, they have found the constrains of $(\lambda, m, \xi_a, \xi_b)$:

$$\xi_a > (1 + 1/\lambda^2)m^2, \qquad \xi_b = 0 \tag{4.8}$$

We can also extend the choice of the gauge charges for the chiral fields under the condition of $(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)$. We write the general gauge charges as in Table 3. The VEV of fields for the vacua can be calculated in the general setting.

Field	$U(1)_a$	$U(1)_b$	$U(1)_R$
Φ_1	$-a$	0	$2/3$
Φ_2	$+a$	$-b$	$2/3$
Φ_3	0	$+b$	$2/3$
Φ_4	$+a$	$+b$	1
Φ_5	$-a$	$-b$	1

Table 3: The field content and charge assignments for Dienes' model for general case.

We list the vacua structure as in Table 4[6]

[6]There are several unstable vacua, here we list only one as an example.

Label	$(\phi_1,\phi_2,\phi_3,\phi_4,\phi_5)$	V	Stability	SUSY	R-symmetry
A	$(\sqrt{\frac{5}{a}},0,0,0,0)$	0	Stable	Yes	Yes
B	$(0,0,\sqrt{\frac{5a-1}{a^2}},0,\sqrt{\frac{5a-1}{a^2}})$	$\frac{10a-1}{2a^2}$	Metastable	No	No
C	$(0,0,0,0,\sqrt{\frac{5a-1}{a^2+b^2}})$	$\frac{(10a-1)(a^2+2b^2)-(1-5a)^2b+25b^2}{2(a^2+b^2)^2}$	Unstable	No	No

Table 4: *The vacuum structure of Dienes' model for general case.*

As we can see, the vacua structure is the same as the original Dienes' model. However, for the metastable state to exist, there are constrains on the gauge charges:

$$\frac{1}{2}(5-\sqrt{21}) < a < \frac{1}{2}(5+\sqrt{21}), \qquad 0 < b \leq 1 \tag{4.9}$$

Inside these constraints, we can have more freedom to choose the charges.

It is interesting to observe that in the above general discussion, some of the U(1) charges of Chiral superfields are chosen to be zero. The reader may ask whether this fact plays any essential role in the model. The answer is without a zero charge there is no supersymmetry vacua any more. As we can see from (4.4), each term in the scalar potential is equal to or larger than zero. As a result if there is no zero U(1) charge in the D-terms, the scalar potential can't be equal to zero. For example, for a scenario with all non-zero charges, the D square part in the potential is as below:

$$\begin{aligned} V_D &= \frac{1}{2}(\xi_a - |\phi_1|^2 + 3|\phi_2|^2 - 2|\phi_3|^2 + |\phi_4|^2 - |\phi_5|^2)^2 \\ &+ \frac{1}{2}(\xi_b - 2|\phi_1|^2 + 5|\phi_2|^2 - 3|\phi_3|^2 + 2|\phi_4|^2 - 2|\phi_5|^2)^2 \end{aligned} \tag{4.10}$$

There is no way we can make both terms equal to zero simultaneously under the conditions of $\xi_a \neq 0$, $\xi_b \neq 0$ and all the U(1) charges are non-zero. We conclude that to have a supersymmetric vacuum requires a prescription:

Choosing one field from ϕ_1, ϕ_2, and ϕ_3 and set its $U_b(1)$ gauge charge to be zero and set the corresponding FI parameter ξ_b to be zero.

As an example, consider a scenario with the U(1) charge chosen, according to our prescription, as in Table 5

The scalar potential becomes

$$\begin{aligned} V &= |\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_4|^2 + |\phi_5|^2 \\ &+ \frac{1}{2}(5 - |\phi_1|^2 + 3|\phi_2|^2 - 2|\phi_3|^2 + |\phi_4|^2 - |\phi_5|^2)^2 \\ &+ \frac{1}{2}(-|\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 - |\phi_5|^2)^2 \ . \end{aligned} \tag{4.11}$$

Field	$U(1)_a$	$U(1)_b$	$U(1)_R$
Φ_1	-1	0	$2/3$
Φ_2	$+3$	-1	$2/3$
Φ_3	-2	$+1$	$2/3$
Φ_4	$+1$	$+1$	1
Φ_5	-1	-1	1

Table 5: The field content and charge assignments for an example.

The only zero charge is the zero $U_b(1)$ charge for ϕ_1. The SUSY vacua is $(\sqrt{5}, 0, 0, 0, 0)$. We can also list its vacua structure as below:

Label	$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$	V	Stability	SUSY	R-symmetry
A	$(\sqrt{5}, 0, 0, 0, 0)$	0	Stable	Yes	No
B	$(0, 0, \frac{4}{3}, 0, \frac{\sqrt{10}}{3})$	$\frac{25}{18}$	Metastable	No	No
C	$(\sqrt{\frac{13}{8}}, 0, \sqrt{\frac{7}{8}}, 0, \frac{\sqrt{3}}{2})$	$\simeq 2.562$	Unstable	No	No

Table 6: *The vacuum structure of an example.*

The surface and contour plots can be obtained in the same way we did in the last section. They can again be used to illustrate the structure of the vacua.

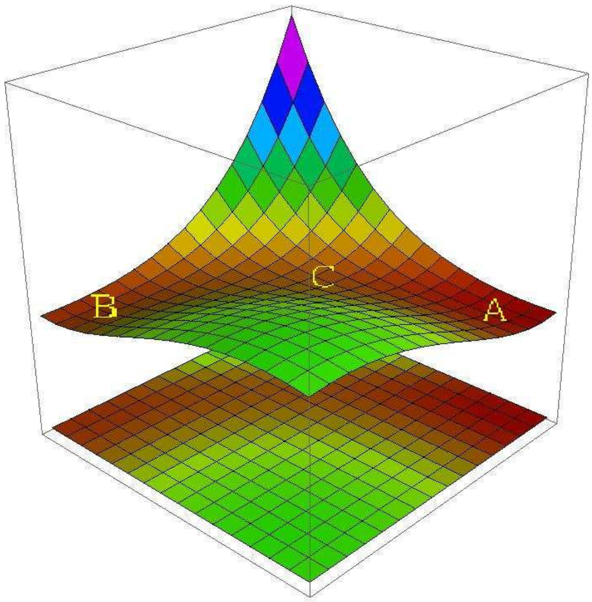

Figure 5: *The surface plot of the scalar potential V on the plane containing points A,B,C vs the VEV's of* (ϕ_1, ϕ_3).

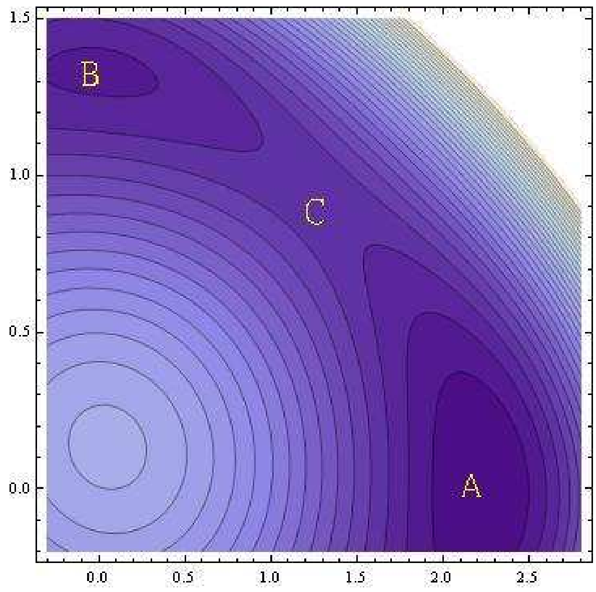

Figure 6: *The metastable vacua and the SUSY vacua are shown.*

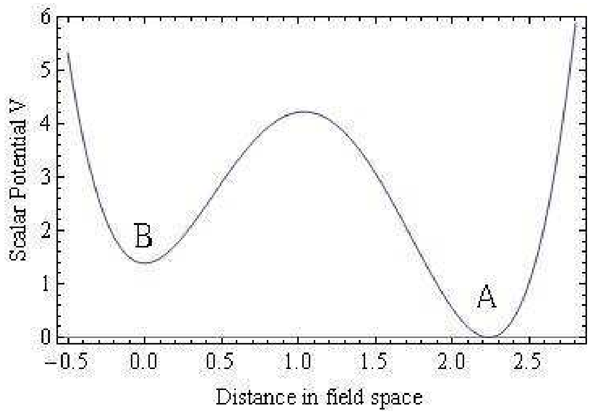

Figure 7: *The scalar potential is illustrated by different points which are on the line between SUSY and metastable vacua.*

4.3 *The life time of metastable vacua*

It is necessary to understand the life time of metastable vacua which must be at least longer than the age of the universe, otherwise the metastable vacua is possible to decay into the SUSY state. Actually the transition rate of a metavacua to the true vacuum occurs through instanton transition. The rate per unit volume is given by [11]:

$$\frac{\Gamma_{inst}}{V_{unit}} = \alpha e^{-\beta} \tag{4.12}$$

Here we only concentrate on the exponent term β which identifies the difference of action between a particular point and metastable vacua in field space: $\beta \equiv S_E(\phi) - S_E(\phi_B)$. In Dienes' paper, they introduced a method [12] which claimed to calculate the β term approximately. The method is based on the ratio of difference of the distance and the potential value between three selected points in field space. Particularly these three points are the supersymmetry, metastable and the unstable vacua which form a triangle. The following notations are defined as $\Delta\phi_B$ is the field distance between the unstable(C) and metastable(B). $\Delta\phi_A$ is the distance between the SUSY vacua(A) and the unstable vacua. Similarly ΔV_B and ΔV_A are the differences of potential of the metastable, and the SUSY with the unstable vacua respectively.

Using the approximation, there is another constant should be identified

$$c \equiv \frac{\Delta V_A}{\Delta V_B}\frac{\Delta\phi_B}{\Delta\phi_A} \tag{4.13}$$

If

$$\frac{\Delta\phi_A}{\Delta\phi_B} \geq \frac{\sqrt{1+c}+1}{\sqrt{1+c}-1} \tag{4.14}$$

then the β can be calculated

$$\beta = \frac{32\pi^2}{3}\frac{1+c}{(\sqrt{1+c}-1)^4}\left(\frac{\Delta\phi_B^4}{\Delta V_B}\right) \tag{4.15}$$

On the other hand, when (4.14) is not satisfied, the β term can be calculated by another method

$$\beta = \frac{\pi^2}{96}\left(\frac{\Delta V_B}{\Delta\phi_B}\right)^2 R_T^3(-\gamma_B^3 + 3c\gamma_B^2\gamma_A + 3c\gamma_A^2\gamma_B - c^2\gamma_A^3) \tag{4.16}$$

where

$$\gamma_i \equiv \left(\frac{8\Delta\phi_i^2}{\Delta V_i}\right)^{1/2}, i = A, B \qquad R_T \equiv \frac{1}{2}\left(\frac{\gamma_B^2 + c\gamma_A^2}{c\gamma_A - \gamma_B}\right) \tag{4.17}$$

Based on the β term calculation, we find the value of β for the metastable vacua in Dienes' model and our example model are greater than 1000, which would implies the metastable lifetime exceed the age of the universe.

5. Extending and simplifying the Dienes' model

The model of Dienes and Thomas is complicated, requiring five chiral fields and two U(1) gauge interactions. Of course, it's precisely this complication that enables a potential twisted enough to afford a rich vacuum structure. But we do want to know how we can simplify the model without ruining the metastable states. The model contains several parameters and features that can be adjusted. Hence we generalize and simplify the model to consider possible extension and demonstrate how the crucial features could be preserved. As we study the Dienes' model in the previous section, we have obtained some insight about the crucial features which can not only break supersymmetry by metastable vacua but also preserve supersymmetry. In this section, we are going try out to extend and simplify the Dienes' model. First, we consider a similar model but with only one gauge group. Then, the number of fields are reduced. Finally we dropped the mass terms in the superpotential. In all cases, we can find scenarios that work as well as the original Dienes' model.

5.1 *Models with only one U(1) gauge group*

There are two U(1) gauge symmetries in the Dienes' model. They serve to restrict the number of interaction terms in the superpotential to a manageable size. However, it's fair to ask if the two U(1) gauge symmetries and their corresponding D terms are really needed in terms of generating the metastable state. After all the superpotential is protected by non-renormalizable theorem. Renormalization won't change the form of the superpotential even if the form is not protected by symmetry as we usually demand in ordinary quantum field theory. That is, we can simply say the superpotential is given as such and it won't be changed after renormalization.

To answer the question of the necessity of two U(1) symmetries, we retain the same superpotential but get rid of one U(1) symmetry.

The U(1) charge and the $U(1)_R$ of each fields are chosen as in Table 7 The superpotential is:

$$W = \lambda\Phi_1\Phi_2\Phi_3 + m\Phi_4\Phi_5 \tag{5.1}$$

We set the parameters as $(\lambda, m, \xi_a, g_a) = (1, 1, 10, 1)$ and the form of superpotential is the same as in the previous section but there is no ξ_b any more because we

Field	$U(1)$	$U(1)_R$
Φ_1	a	$2/3$
Φ_2	b	$2/3$
Φ_3	$-(a+b)$	$2/3$
Φ_4	c	1
Φ_5	$-c$	1

Table 7: *The field content and charge assignments for the model with only one $U(1)$ gauge group.*

only introduce one gauge group. The scalar potential can then be written as below

$$\begin{aligned} V &= |\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_4|^2 + |\phi_5|^2 \\ &+ \frac{1}{2}(10 + a|\phi_1|^2 + b|\phi_2|^2 - (a+b)|\phi_3|^2 + c|\phi_4|^2 - c|\phi_5|^2)^2 \end{aligned} \tag{5.2}$$

We assume the general case of unspecified U(1) charges (a, b, c) and then search for the extreme values of the potential:

$$\frac{\partial V}{\partial \phi_i} = 0 \qquad i = 1\ldots 5 \tag{5.3}$$

For the extrema to be stable, they must satisfy the condition that the mass matrix contains only non-negative eigenvalues. The number of zero eigenvalues must equal the number of Goldstone bosons. Here the number of zero is 1.

$$\mathcal{M}^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial\phi_i \partial\phi_j} & \frac{\partial^2 V}{\partial\phi_i \partial\phi_j^*} \\ \frac{\partial^2 V}{\partial\phi_i^* \partial\phi_j} & \frac{\partial^2 V}{\partial\phi_i^* \partial\phi_j^*} \end{pmatrix} \tag{5.4}$$

Indeed we can find a vacuum structure similar as in the Dienes' model with one supersymmetric, one metastable and several unstable vacua. We even obtain the range of gauge charge (a, b, c) for this to work .

$$a < 0, \qquad |a| < b, \qquad c > \frac{1}{10}(1 - 2a^2 - 2ab) \tag{5.5}$$

As an illustration, we choose the U(1) charges, under the above condition, as in Table 8 The scalar potential now is

$$\begin{aligned} V &= |\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_4|^2 + |\phi_5|^2 \\ &+ \frac{1}{2}(10 - |\phi_1|^2 + 3|\phi_2|^2 - 2|\phi_3|^2 + 2|\phi_4|^2 - 2|\phi_5|^2)^2 \end{aligned} \tag{5.6}$$

Field	$U(1)$	$U(1)_R$
Φ_1	-1	$2/3$
Φ_2	$+3$	$2/3$
Φ_3	-2	$2/3$
Φ_4	$+2$	1
Φ_5	-2	1

Table 8: *The chosen gauge charge assignments for the model with only one $U(1)$ gauge group.*

and by solving (4.6) and satisfy the non-negative eigenvalues of (5.4). We can obtain the vacuum structure which contains supersymmetric, metastable and unstable vacua in Table 9

Label	$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$	V	Stability	SUSY	R-symmetry
A	$(\sqrt{10}, 0, 0, 0, 0)$	0	Stable	Yes	Yes
B	$(1, 0, \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{15}}{2})$	$\frac{35}{8}$	Metastable	No	No
C	$(0, 0, 0, 0, \frac{\sqrt{19}}{2})$	$\frac{39}{8}$	Unstable	No	No

Table 9: *The vacuum structure of the model.*

We draw the plots of the potential in field space as we did in the previous section.

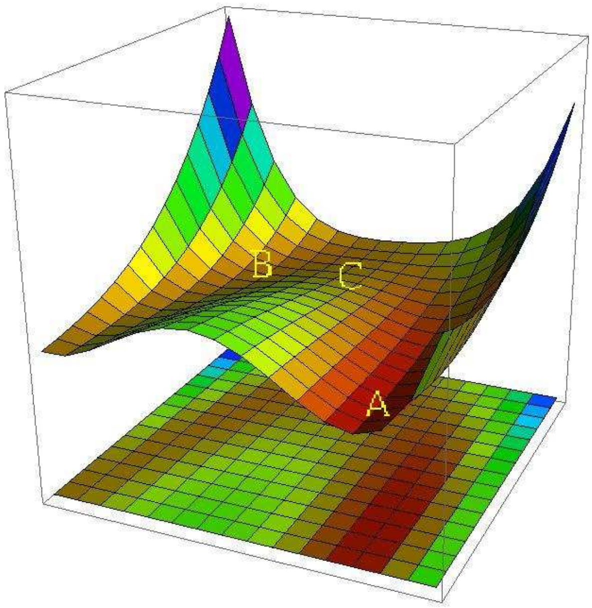

Figure 8: *The surface plot of scalar potential V on the plane containing points A,B,C vs the VEV's of* (ϕ_1, ϕ_3).

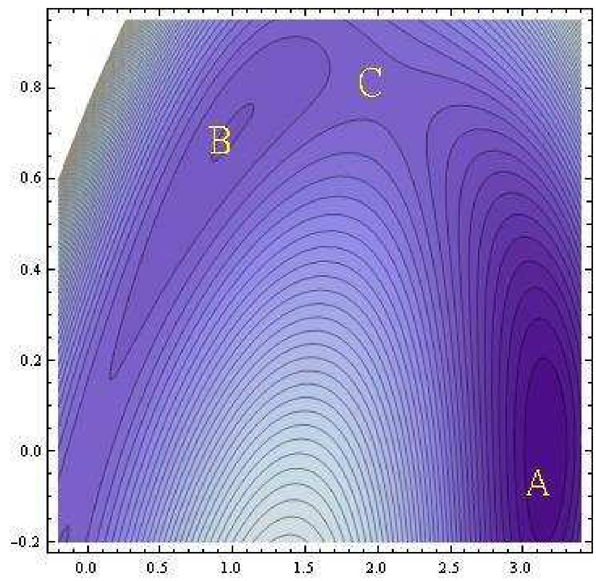

Figure 9: *The contour plot.*

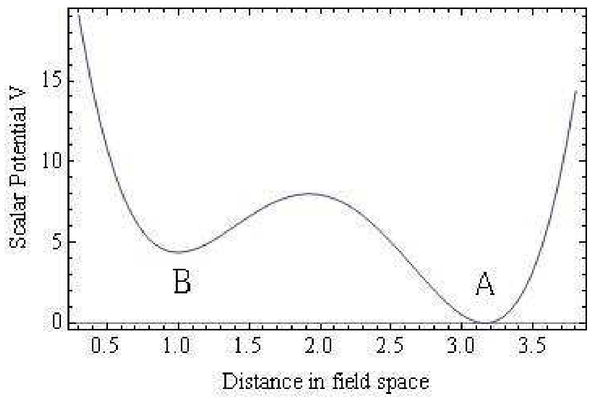

Figure 10: *The scalar potential is illustrated by different points which are on the line between SUSY and metastable vacua.*

Now we consider the general case for parameters.

$$W = \lambda\Phi_1\Phi_2\Phi_3 + m\Phi_4\Phi_5 \tag{5.7}$$

and

$$\begin{aligned} V &= \lambda^2(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2) + m^2(|\phi_4|^2 + |\phi_5|^2) \\ &\quad + \frac{1}{2}(\xi + a|\phi_1|^2 + b|\phi_2|^2 - (a+b)|\phi_3|^2 + c|\phi_4|^2 - c|\phi_5|^2)^2 \end{aligned} \tag{5.8}$$

Now we allow all parameters to vary. The conditions of the gauge charge(a, b, c) and the parameters(λ, m, ξ) for the model to contain both supersymmetry vacua and metastable vacua are as following:

$$\begin{aligned} &a < 0, \quad b > -a, \quad c > 0, \quad m > 0, \quad \xi > \frac{m^2}{c}, \quad \lambda > \sqrt{\frac{2m^2(a^2+ab)}{m^2 - c\xi}} \\ &\qquad (or \quad m < 0, \quad \xi > \frac{m^2}{c}, \quad \lambda > \sqrt{\frac{2m^2(a^2+ab)}{m^2 - c\xi}}) \end{aligned} \tag{5.9}$$

or

$$\begin{aligned} &a > 0, \quad b < -a, \quad c < 0, \quad m > 0, \quad \xi < \frac{m^2}{c}, \quad \lambda > \sqrt{\frac{2m^2(a^2+ab)}{m^2 - c\xi}} \\ &\qquad (or \quad m < 0, \quad \xi > \frac{m^2}{c}, \quad \lambda > \sqrt{\frac{2m^2(a^2+ab)}{m^2 - c\xi}}) \end{aligned} \tag{5.10}$$

For both (5.9) and (5.10), there are another ranges existed for the negative sign of λ. For simplicity, we only concentrate on the positive sign of λ.

5.2 *Models with fewer chiral superfields*

In this subsection, we continue to simplify Dienes' model to involve fewer fields while keeping only one U(1) gauge group. Again we first write the general case:

$$W = \lambda\Phi_1\Phi_2\Phi_3 + m\Phi_1\Phi_4 \tag{5.11}$$

and the U(1) charges of each field as showed in Table 10.

Field	$U(1)$	$U(1)_R$
Φ_1	$+a$	$2/3$
Φ_2	$+b$	$2/3$
Φ_3	$-(a+b)$	$2/3$
Φ_4	$-a$	$4/3$

Table 10: *The field content and charge assignments for the 4-field model with only one $U(1)$ gauge group.*

The scalar potential can be obtained straightforwardly

$$\begin{aligned} V &= \lambda^2(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2) + m^2(|\phi_1|^2 + |\phi_4|^2) \\ &\quad +2m\lambda\phi_2\phi_3\phi_4 + \frac{1}{2}(\xi + a|\phi_1|^2 + b|\phi_2|^2 - (a+b)|\phi_3|^2 + a|\phi_4|^2)^2 \end{aligned} \tag{5.12}$$

Again we find the potential has a SUSY breaking metastable state in addition to a SUSY vaccum. The range of parameters space for this to work is:

$$a < 0, \qquad b > -a, \qquad \xi > \frac{-m^2}{a}, \qquad \lambda > \sqrt{\frac{m^2(a^2+ab)}{m^2+a\xi}}, \qquad m \in R \setminus \{0\} \tag{5.13}$$

or

$$a > 0, \qquad b < -a, \qquad \xi < \frac{-m^2}{a}, \qquad \lambda > \sqrt{\frac{m^2(a^2+ab)}{m^2+a\xi}}, \qquad m \in R \setminus \{0\} \tag{5.14}$$

Here there is also the range for $\lambda < -\sqrt{\frac{m^2(a^2+ab)}{m^2+a\xi}}$. Again for simplicity, we only focus on positive range.

As an example we choose, according to (5.13) and (5.14), a scenario with the parameters as $(\lambda, \xi, m) = (1, 10, 1)$ and the U(1) charges as in Table 11

The scalar potential becomes

$$\begin{aligned} V &= |\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_1|^2 + |\phi_4|^2 \\ &\quad +2\phi_2\phi_3\phi_4 + \frac{1}{2}(10 - |\phi_1|^2 + 3|\phi_2|^2 - 2|\phi_3|^2 + |\phi_4|^2)^2 \end{aligned} \tag{5.15}$$

and the vacua structure is illustrated in Table 12

Field	$U(1)$	$U(1)_R$
Φ_1	-1	$2/3$
Φ_2	$+3$	$2/3$
Φ_3	-2	$2/3$
Φ_4	$+1$	$4/3$

Table 11: *The field content and charge assignments for the 4-field model with only one $U(1)$ gauge group.*

Label	$(\phi_1, \phi_2, \phi_3, \phi_4)$	V	Stability	SUSY	R-symmetry
A	$(0, 0, \sqrt{5}, 0)$	0	Stable	Yes	Yes
B	$(3, 0, 0, 0)$	$\frac{19}{2}$	Metastable	No	No
C	$(\sqrt{\frac{24}{5}}, 0, \sqrt{\frac{7}{5}}, 0)$	$\frac{72}{5}$	Unstable	No	No

Table 12: *The vacuum structure of the 4-field model.*

We draw the plots of the potential in field space as we did in the previous section.

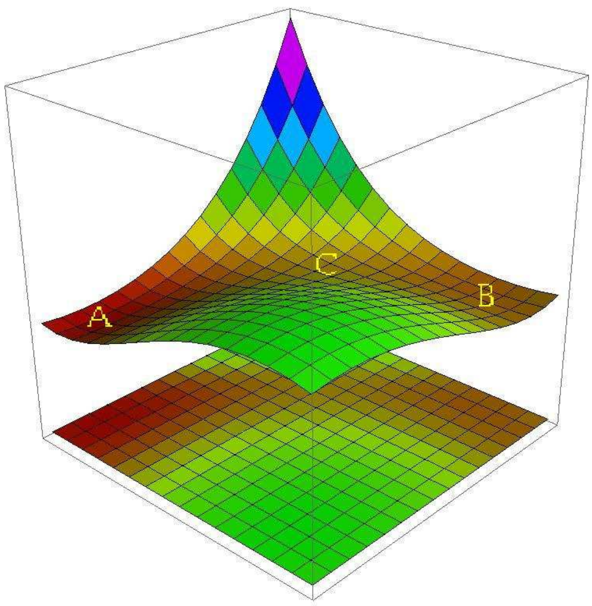

Figure 11: *The surface plot of scalar potential V of 4-field model scalar potential V on the plane containing points A,B,C vs the VEV's of (ϕ_1, ϕ_3).*

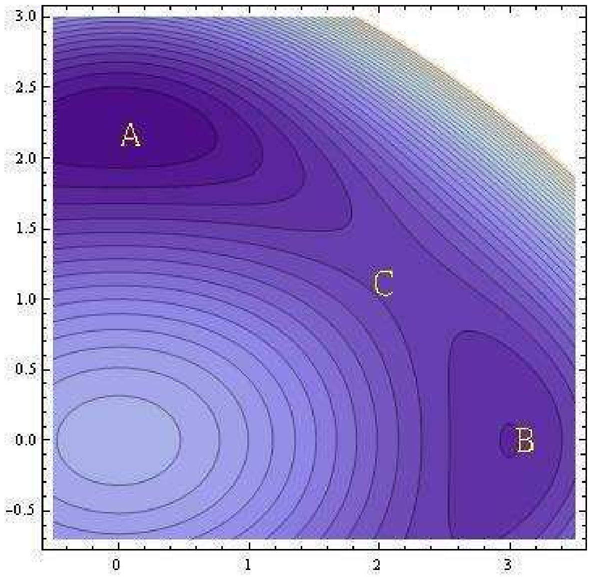

Figure 12: *The contour plot.*

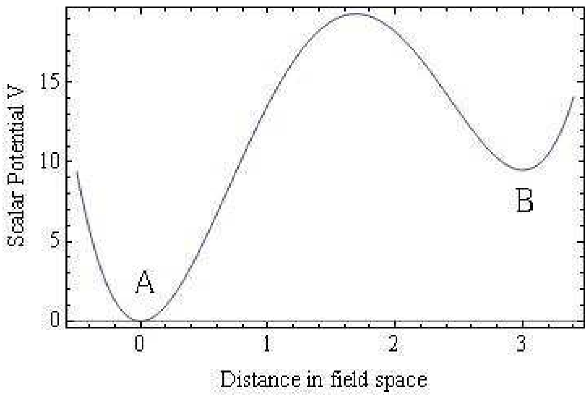

Figure 13: *The scalar potential is illustrated by different points which are on the line between SUSY and metastable vacua.*

5.3 *Models with no mass terms in the superpotential*

Now we have shown it is possible to reduce the number of fields and U(1) gauge symmetries in Dienes' model without ruining its metastable SUSY breaking state. In these simplified versions we retain almost the same superpotential with a cubic term and a mass quadratic term. The mass term introduces a dimensionful parameter in addition to the FI term coefficient. It's clear now FI term is necessary. We may ask if we can avoid one more scale in the model by dropping the mass term. It is implied in [6] that mass term is necessary. Indeed simply dropping mass term will render too simple a scalar potential to give a metastable state. However we can consider a scenario in which the superpotential contains more than one cubic terms.

In this model, there are four chiral superfields. Its superpotential is

$$W = \lambda_1 \Phi_1^2 \Phi_2 + \lambda_2 \Phi_1 \Phi_3 \Phi_4. \tag{5.16}$$

The U(1) and $U(1)_R$ charges of each fields are given as in Table. 13

Field	$U(1)$	$U(1)_R$
Φ_1	a	$2/3$
Φ_2	$-2a$	$2/3$
Φ_3	b	$2/3$
Φ_4	$-(a+b)$	$2/3$

Table 13: *The field content and charge assignments for the model with no mass term.*

From two cubic terms in the superpotential, the scalar potential is not too simple.

$$\begin{aligned} V &= \lambda_1^2(4|\phi_1|^2|\phi_2|^2 + |\phi_1^2|^2) + \lambda_2^2(|\phi_1|^2|\phi_3|^2 + |\phi_1|^2|\phi_4|^2 + |\phi_3|^2|\phi_4|^2) \\ &\quad +4\lambda_1\lambda_2\phi_1\phi_2\phi_3\phi_4 + \frac{1}{2}(\xi + a|\phi_1|^2 - 2a|\phi_2|^2 + b|\phi_3|^2 - (a+b)|\phi_4|^2)^2 \end{aligned} \tag{5.17}$$

We find extrema of (5.17) and check the stability condition and obtain the vacua structure[7] as in Table. 15

Label	$(\phi_1, \phi_2, \phi_3, \phi_4)$	V	Stability	SUSY	R-symmetry
A	$(0, 0, \sqrt{-b\xi}, 0)$	0	Stable	Yes	Yes
B	$(\sqrt{\frac{-a\xi}{2\lambda_1^2+a^2}}, 0, 0, 0)$	$\frac{\lambda_1^2\xi^2}{2\lambda_1^2+a^2}$	Metastable	No	No
C	$(\sqrt{\frac{b\xi\lambda_2^2}{-2\lambda_2^4-3ab\lambda_2^2+2b^2\lambda_1^2}}, 0, \sqrt{\frac{2\xi(b\lambda_1^2-a\lambda_2^2)}{2\lambda_2^4+3ab\lambda_2^2-2b^2\lambda_1^2}}, 0)$	$*$	Unstable	No	No

Table 14: *The vacuum structure of the model which only involves interaction terms*

Similarly, we can solve the range of parameters space in which the scalar potential contains a SUSY vacuum and a metastable SUSY breaking state as follows:

$$a < 0, \qquad \frac{a\lambda_2^2}{2\lambda_1^2} < b < 0, \qquad \xi > 0, \qquad \lambda_1 > 0, \qquad \lambda_2 > \sqrt{\frac{2b}{a}}\lambda_1 \tag{5.18}$$

or

$$a > 0, \qquad 0 < b < \frac{a\lambda_2^2}{2\lambda_1^2}, \qquad \xi < 0, \qquad \lambda_1 > 0, \qquad \lambda_2 > \sqrt{\frac{2b}{a}}\lambda_1 \tag{5.19}$$

[7] $* = \frac{\xi^2\lambda_2^4[2(\lambda_2^4+ab\lambda_2^2)-b^2\lambda_1^2]}{(2\lambda_2^4+3ab\lambda_2^2-2b^2\lambda_1^2)^2}$

Here we simplify the constraint by setting λ_1 and λ_2 both positive. In addition, the signs of charge of Φ_3 and Φ_4 are opposite. We have a freedom to switch their gauge charges since they play equal roles in superpotential.

As a simple example, we can choose $(\lambda_1, \lambda_2, \xi) = (1, 2, 6)$ and the gauge charges are chosen as below:

Field	$U(1)$	$U(1)_R$
Φ_1	-1	$2/3$
Φ_2	$+2$	$2/3$
Φ_3	-1	$2/3$
Φ_4	$+2$	$2/3$

Table 15: *The field content and charge assignments for the model with no mass term.*

So the scalar potential is

$$\begin{aligned} V &= 4|\phi_1|^2|\phi_2|^2 + |\phi_1^2|^2 + 4(|\phi_1|^2|\phi_3|^2 + |\phi_1|^2|\phi_4|^2 + |\phi_3|^2|\phi_3|^2) \\ &\quad +8\phi_1\phi_2\phi_3\phi_4 + \frac{1}{2}(6 - |\phi_1|^2 + 2|\phi_2|^2 - |\phi_3|^2 + 2|\phi_4|^2)^2 \end{aligned} \tag{5.20}$$

and the vacua structure turns to Table 16.

Label	(ϕ_1, ϕ_2, ϕ_3)	V	Stability	SUSY	R-symmetry
A	$(0, 0, \sqrt{6}, 0)$	0	Stable	Yes	Yes
B	$(\sqrt{2}, 0, 0, 0)$	12	Metastable	No	No
C	$(\sqrt{\frac{24}{23}}, 0, \sqrt{\frac{18}{23}}, 0)$	$\simeq 13.066$	Unstable	No	No

Table 16: *The vacuum structure of simple example of this model.*

Consequently, we have built a model without mass term.

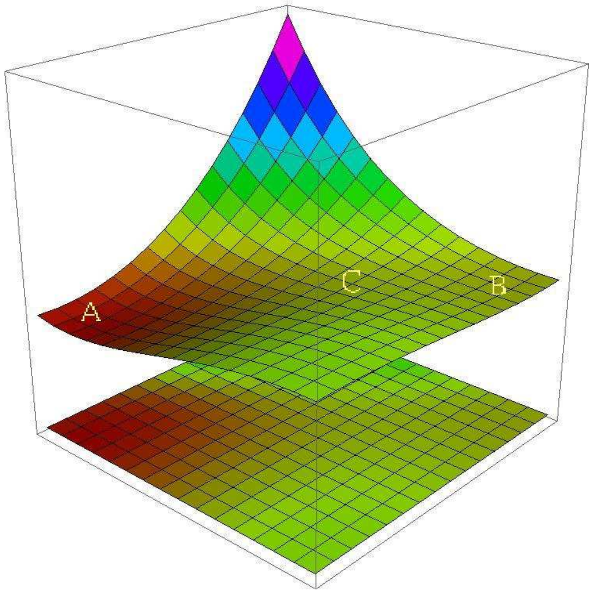

Figure 14: *The surface plot of scalar potential V of only interaction model in the special field space which is constructed by two fields* (ϕ_1, ϕ_3).

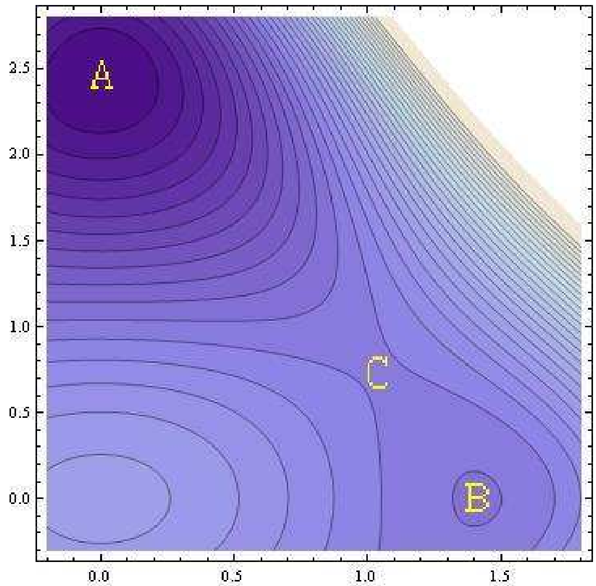

Figure 15: *The supersymmetry and metastable vacua are showed on the top left side and right bottom respectively.*

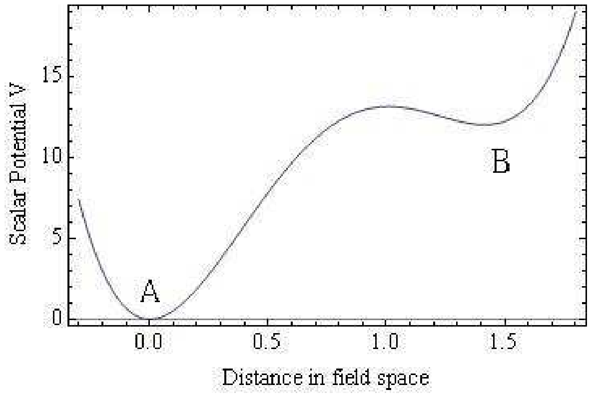

Figure 16: *The scalar potential is illustrated by different points which are on the line of SUSY and metastable vacua.*

We can discuss the lifetime of metastable state of this model. According to previous section, we have understood the relation between lifetime exponent term and difference of potential and distance in field space. Here we concentrate on the effect of parameters λ_1 and λ_2. For simplification reason, we set λ_1 to be 1 and we have know the constrain of $\lambda_2 > \sqrt{2}\lambda_1$. Therefore we can plot the relation graph as below:

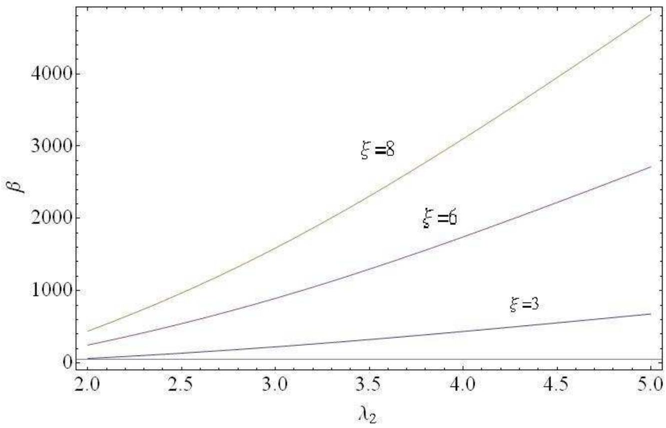

Figure 17: *The exponent term of lifetime of metastable vacua is plotted as a function of λ_2 for different ξ values. Here we set λ_1 to be 1. The horizontal line which is near the button frame axes line indicates 50 of β value for reference reason.*

As the Figure 17 shown, we can choose the proper λ_2 and ξ to expand the lifetime of our metastable state to let it longer than the age of the universe.

In addition, the superpotential can be added by other possible terms which are all interaction terms. Actually (5.16) comes from

$$W = (\lambda_1\Phi_1 + \lambda_2\Phi_3)^2\Phi_2 + (\lambda_3\Phi_1 + \lambda_4\Phi_3)^2\Phi_4 \tag{5.21}$$

However, this complete superpotential can not consist of the SUSY vacua because it leads to the existence of $|\phi_3^2|^2$ term which should be zero in the scalar potential to preserve SUSY in (5.17). As a result, there is no SUSY vacuum anymore. About the metastable vacua, it will disappear since there is a $\phi_1^3\phi_3$ term in the scalar potential.

5.4 *Summary*

Based on previous models, we have noticed some intriguing feature about the SUSY vacua. The SUSY vacua has the strong relationship with the ξ in D-term. As we can see, if we'd like to solve the SUSY vacua, we must to control the expecting value of chiral field to let both F-term and D-term to be zero. As a result, if we choose one field to let D-term to be zero and the remaining fields are all zero. For example, in (5.15) and (5.20) the Φ_3 plays the role to let D-term to be zero. Secondly the remaining fields play the roles to combine with Φ_3 in F-term. As long as we let these fields to be zero and we can have the SUSY vacua eventually. As the SUSY vacua in Table 12 and Table 15, the expectation value of Φ_3 are exactly equal to $\sqrt{\xi}$ and the remaining fields are combined with Φ_3 and they are all zero value in order to form the SUSY vacua. Therefore we do not need to choose one field to set its U(1) charge to be zero as what we claimed in Section 4.

About the metastable vacua, it is not as determined as SUSY vacua even though we have observed that metastable vacua may appear in the field which has the same sign U(1) charge as the SUSY vacua field has. In models of (5.15) and (5.20), Φ_1 has the same U(1) charge as Φ_3 which is the SUSY vacua field in both models. However it does not guarantee that Φ_1 is the metastable vacua field and it must be satisfied the conditions of (5.3), non-negative eigenvalues of mass matrix and the proper number of Goldstone boson to break U(1) symmetry if necessary.

Finally, the lifetime of all metastable states in the above mentioned models are greater than the age of the university through the proper choice of values for the parameters such as λ and ξ.

6. Conclusion

In this thesis we discussed two cases of perturbatively realized SUSY breaking by metastable states. In both scenarios, the vacuum structure can be studied by tree level calculations. They are shown to contain similar vacua structures with a true SUSY vacuum, a metastable SUSY breaking vacuum and some unstable vacua.

We start with the O'Raifeartaigh models and study the modified cases allowing for more general superpotential. The generic functions in the superpotential are modified to study their effects on the vacuum stucture. We especially observe the effect of changing the order of generic functions in superpotential and obtain constraints. We also calculate the spectrum of fermions and bosons in the models.

In the second part of this thesis, we move to study a model proposed by Dienes and Thomas. It is basically a hybrid of Wess-Zumino model and Fayet-Illiopolous model, with no-zero FI terms for the U(1) Abelian Gauge symmetries. The scalar potential of such a model is complicated enough to allow for a metastable SUSY breaking vacuum. We concentrate on building simpler model which contains the same vacua structure as Dienes' model. It is not quite easy to build a model that works as Dienes' model does. After all it's the complexity that provides a possibility for metastable states. After several failed attempt, it turns out we can get away with only one U(1) gauge interaction. In some cases, we can even reduce the number of chiral superfields. We study these possibilities in details and obtain constraints on the parameters.

It was claimed in Dienes' paper that a mass term in the superpotential is necessary for the local stability of the metastable state. Nevertheless we have successfully built a model without mass terms that also provides metastable states. This step broadens the possibility of model building. The vacuum structure arises from the interplay between two cubic terms in the superpotential and the U(1) FI term. The FI term provides the scale for SUSY breaking.

From the above investigations, it seems that perturbative metastable SUSY breaking is quite generic. Though the scalar potential needs to be complicated enough, the needed complexity is not overwhelming. This opens up a whole lot of model building possibilities. Their implications for experimental observable remains to be studied.

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