



NATIONAL TAIWAN NORMAL UNIVERSITY

Higgs Production by Gluon Fusion in New Physics

Lin, Da Wei

supervised by
Dr. Chen, Chuan-Ren BROWN

August 17, 2016

Abstract

The existence of the Standard Model Higgs boson was verified by CERN experiments, with its mass found to be 125 GeV. Its production cross section by the dominant gluon-gluon fusion channel can be readily written down by making use of low-energy theorems, which are valid in or beyond the Standard Model. In this thesis the effects of exotic fermions or scalars to Higgs productions are considered. In these models, there would be additional contributions to the Higgs production amplitude by Feynman diagrams in which the top quark loops in the gluon-gluon fusion process are replaced by these exotic particle loops. Similar assertions hold for the two gamma decay channel of the Higgs boson, by means of which the predictions of the models can be checked, if statistics improves in future experiments.



keywords: Higgs Production, Gluon Fusion, New Physics

Acknowledgements

I want to thank all of NTNU professors such as Prof. Chen, Chuan-Ren, Prof. Sze, Wah-Keung and so on. Every one professor taught me the particle physics concept and other basic physics concepts. My advisor taught me how to research the particle physics and gave me the research topics that was the Higgs production by Gluon fusion in new physics. Prof. Sze taught me how to write the graduate paper and what kind of paper format that was the good paper. In short, I'm very happy in graduate life and meet NTNU professors. Thank all of NTNU professor.



Contents

Abstract	i
Acknowledgements	ii
1 Introduction	2
2 The Lagrangian Formulation of Dynamical Systems	3
2.1 Hamilton's Principle	4
2.2 Continuous Systems	4
2.3 Lorentz Covariant Field Theory	5
2.4 Complex Scalar Field	6
3 Spontaneous Symmetry Breaking	7
3.1 Spontaneous Breaking of Global Symmetries	8
3.2 Spontaneous Breaking of Local Symmetries	10
4 Higgs Particle in the SM	12
4.1 The SM before Electroweak Symmetry Breaking	12
4.2 The Higgs Mechanism in the SM	15
5 Low Energy Theorem	19
6 Phenomenology	21
6.1 Higgs Production by Gluon-Gluon Fusion in SM	22
6.2 Low Energy Theorem in New Physics	23
7 Conclusion	37
8 Bibliography	39
A Function A	41

Chapter 1

Introduction

The Higgs bosons are discovered at 125 GeV by CERN experiment. Higgs production in the Standard Model (SM) has many kind of production methods such as gluon-gluon fusion, vector boson fusion, ttH process and bbH process. This paper only considers gluon-gluon mechanism for Higgs bosons when Higgs boson is at 125 GeV. The Higgs production by gluon-gluon fusion is produced by proton-proton collider experiment. The Feynman diagram of the Higgs production by gluon-gluon fusion in SM only considers the top quark loops because the events and particle mass have the relations. According the low-energy theorem for Higgs bosons, the cross section of the Higgs production by gluon-gluon fusion only considers at leading order term in SM. Moreover, Top quark loops are replaced by new particle loops in new physics. We can define a new parameter R_i which describes new physics phenomenology. R_i is defined by new cross section over SM cross section. If the extra new particles contribute to the Higgs production by gluon fusion in SM and exists, the new cross section will be bigger or smaller than the SM cross section. It will have two cases such as fermion case and color scalar case. In addition, decay part only considers two gamma decay width of the Higgs boson which has top quark loop and W boson loop in SM. W loop or top quark loop is replaced by new quark loops in new physics decay process. We also can defined a new parameter R_Γ which describes new physics decay process. R_Γ is defined by new decay width over SM decay width. In short, all of case will contribute to two gamma decay width of the Higgs boson in new physics. The proton distribution function (PDF) database will be used MSTW2008lo68cl database of LHAPDF. Other new database is base on MSTW2008lo68cl database in LHAPDF. All of calculation process will be used HIGLU program and HDECAY program in Fortran.

Chapter 2

The Lagrangian Formulation of Dynamical Systems

This section will introduce dynamical systems by Lagrangian so there are two part such as Hamilton's principle, continuous systems.



2.1 Hamilton's Principle

In classical theory, Hamilton's principle explains to use the Lagrangian concepts to describe the mechanical systems. Given Lagrangian $L(q, \dot{q}) = T - V$, the action S is defined by

$$S = \int_{t_1}^{t_2} L(q, \dot{q}) dt$$

where $q(t)$ depend on time in general coordination from t_1 to t_2 . Moreover, T is the kinetic energy of the system. V is the potential energy of the system. The value of the action S depends on the path of the integration in the time-space. The action S of Hamilton's principle is stationary for the particular path that is determined by the motion equations so δS is zero.

$$\delta S = \delta \int_{t_1}^{t_2} L(q, \dot{q}) dt = \int_{t_1}^{t_2} \sum \left[\frac{\partial L}{\partial q} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) \right] \delta q dt \quad (2.1)$$

2.2 Continuous Systems

The Hamilton's principle extend to explain the continuous system of the physics. $q(t)$ is replaced by $q(x, t)$. In continuous systems, Hamilton's principle can be written

$$L = T - V = \int_0^l \mathcal{L} dx \quad (2.2)$$

where $\mathcal{L} = \mathcal{T} - \mathcal{V}$ is called the Lagrangian density, with \mathcal{T} and \mathcal{V} being the kinetic and potential energy densities, respectively. The corresponding action S is

$$S = \int_{t_1}^{t_2} dt \int_0^l \mathcal{L}(\dot{\phi}, \phi') dx \quad (2.3)$$

where $\dot{\phi} = \partial\phi/\partial t$ and $\phi' = \partial\phi/\partial x$. Hamilton's principle by Lagrangian density is also stationary for the surface that describes the equation of the motion from initial displacement to final displacement. The δS can be written

$$\delta S = \int_0^l dx \int_{t_1}^{t_2} dt \left[\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \delta(\dot{\phi}) + \frac{\partial \mathcal{L}}{\partial \phi'} \delta(\phi') \right] \quad (2.4)$$

where $\delta(\dot{\phi}) = \partial(\delta\phi)/\partial t$ and $\delta(\phi') = \partial(\delta\phi)/\partial x$. The boundary conditions are zero since

$$\delta\phi(x, t_1) = \delta\phi(x, t_2) = 0 \text{ for all } x$$

and

$$\delta\phi(0, t) = \delta\phi(l, t) = 0 \text{ for all } t.$$

The δS is zero by the action concept of the Hamilton's principle

$$\delta S = \int_0^l dx \int_{t_1}^{t_2} dt \left[-\frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - \frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \phi'} \right) \right] \delta\phi = 0 \quad (2.5)$$

so

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - \frac{\partial}{\partial x} \left(\frac{\partial \mathcal{L}}{\partial \phi'} \right) = 0.$$

The Lagrangian describes the momentum density in the mechanical system so the momentum density can be defined by

$$\mathcal{P} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

from the δS form. The Hamiltonian density describe the continuous system and is also defined by \mathcal{P} and \mathcal{L} .

$$\mathcal{H} = \mathcal{P}\dot{\phi} - \mathcal{L}.$$

When the system is a continuous system, the system is described by Hamilton's principle. The Hamilton's principle can be written

$$H = \int \mathcal{H} dx = \int (\mathcal{P}\dot{\phi} - \mathcal{L}) dx$$

when the Lagrangian density doesn't depend on time.

2.3 Lorentz Covariant Field Theory

The action of the Hamilton's principle adds the Lorentz invariant concepts so the volume element is d^4x in the 4 dimension time-space. The Lagrangian density of the Lorentz invariant also adds the Lorentz invariant concepts in the 4 dimension time-space so the Lagrangian density of the Lorentz invariant can be written

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

when ϕ is a scalar field. The displacement of the field theory is the local concepts so there isn't action at distance. In field theories, there is the field variance at boundary or in the integral paths. The Lagrangian density of the

Lorentz invariant also satisfies the action principle when δS is zero, but if the Lorentz invariant Lagrangian density doesn't consider at the large distance, the Lagrangian density can be written

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = 0. \quad (2.6)$$

2.4 Complex Scalar Field

When the field is the complex scalar fields, the field function is defined by

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

where ϕ_1 and ϕ_2 are real scalar fields. The complex scalar field and the real scalar field must satisfies the Klein-Gordon equation. The Klein-Gordon equation describes the spinless particles in particle physics and is defined by

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

when ϕ is the real scalar fields. The Lagrangian density in the complex scalar field situation can be written[1]

$$\mathcal{L} = g^{\mu\nu} \partial_\mu \phi^\dagger \partial_\nu \phi - m^2 \phi^\dagger \phi = \frac{1}{2}[\partial_\mu \phi_1 \partial^\mu \phi_1 - m^2 \phi_1^2] + \frac{1}{2}[\partial_\mu \phi_2 \partial^\mu \phi_2 - m^2 \phi_2^2] \quad (2.7)$$

where

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Chapter 3

Spontaneous Symmetry Breaking

In symmetry breaking classical system, the ground state only has one point at $\langle 0|\phi^\dagger\phi|0\rangle = 0$, but in spontaneous symmetry breaking system, the ground state of the system has two points when $\langle 0|\phi^\dagger\phi|0\rangle \neq 0$. This section will be introduced spontaneous symmetry breaking by global symmetry and local symmetry. It also is introduced goldstone mechanism by spontaneous breaking of global symmetries in abelian case and non-abelian case, and Higgs mechanism by spontaneous breaking of local symmetries in abelian case.

3.1 Spontaneous Breaking of Global Symmetries

If ϕ is the constant, independent time-space, the ϕ only contributes to the potential energy of the system in the Lagrangian density form. If m^2 is the positive, the Lagrangian density will have the minimum energy when ϕ_1 and ϕ_2 are zero in the complex scalar field situation. When the Lagrangian density has the minimum ground state v that is the real parameter, is also called the vacuum state, the potential energy term in the Lagrangian density form should be written

$$\mathcal{V} = \frac{m^2}{2v^2}(\phi^\dagger\phi - v^2)^2$$

where v is the possible vacuum state in the Lagrangian density of the system. When the Lagrangian density considers the global U(1) symmetry, the field of transformation should be written

$$\begin{aligned} \phi &\rightarrow \phi' = e^{i\theta}\phi; & \mathcal{L} &\rightarrow \mathcal{L}' = \mathcal{L} \\ \begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \end{aligned}$$

when the field is the complex scalar field. If the vacuum state is a real field, the field form is $(v, 0)$ in (ϕ_1, ϕ_2) space. The Lagrangian density adds the extra fields when the vacuum state is real field so the Lagrangian density becomes

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{free} + \mathcal{L}_{int} \\ \mathcal{L} &= \frac{1}{2} \partial_\mu\chi \partial^\mu\chi + \frac{1}{2} \partial_\mu\psi \partial^\mu\psi - \frac{m^2}{2v^2} [\sqrt{2}v\chi + \frac{\chi^2}{2} + \frac{\psi^2}{2}]^2 \end{aligned} \quad (3.1)$$

when $\phi = v + \frac{1}{2}(\chi + i\psi)$. The \mathcal{L}_{free} interprets the free particle fields. The \mathcal{L}_{int} interprets interaction terms.

$$\mathcal{L}_{free} = \frac{1}{2} \partial_\mu\chi \partial^\mu\chi + \frac{1}{2} \partial_\mu\psi \partial^\mu\psi - m^2\chi^2$$

The χ is interpreted as a scalar spinless particles which mass $\sqrt{2}m$. The ψ is also interpreted scalar particle, but it is massless. It is called the Goldstone boson, which necessarily arises in spontaneous breaking of global symmetry, which is elucidated below.

The Goldstone Mechanism

This part will interpret more detail about the Goldstone mechanism. It will start from real scalar field concepts. In the Goldstone situation, the Lagrangian density is defined by

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mathcal{V}(\phi).$$

when the field is the scalar field. The potential of the system in the Goldstone situation is

$$\mathcal{V}(\phi) = \mu^2 \phi^\dagger \phi + \frac{1}{4} \lambda (\phi^\dagger \phi)^2$$

where μ^2 is the mass term, λ is the self-coupling parameter. The potential of system has the minimum states at $\langle 0 | \phi^\dagger \phi | 0 \rangle$ when $\mu^2 < 0$. The minimum state is called the vacuum expectation value of the scalar field:

$$\frac{\partial \mathcal{V}}{\partial \phi} = 2\mu^2 \phi + \lambda \phi^3 = 0$$

$$\phi_0^2 = v^2 = -\frac{2\mu^2}{\lambda}$$

When the field is set by $\phi = v + \sigma$, the Lagrangian density becomes

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - (-\mu^2) \sigma^2 - \sqrt{-\mu^2 \lambda} \sigma^3 - \frac{\lambda}{4} \sigma^4$$

The mass is $-2\mu^2$ when the scalar field of the Lagrangian density is $\phi = v + \sigma$. The σ^3 term and the σ^4 term are the self-interaction terms.

If symmetry group considers $O(4)$ group in non-abelian case, goldstone mechanism will be found out three pions and one σ boson so when the complex scalar field has four scalar fields, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - (-\mu^2) \sigma^2 - \sqrt{-\mu^2 \lambda} \sigma^3 - \frac{\lambda}{4} \sigma^4 + \frac{1}{2} \partial_\mu \pi_i \partial^\mu \pi_i - \frac{\lambda}{4} (\pi_i \pi_i)^2 - \lambda v \pi_i \pi_i \sigma - \frac{\lambda}{2} \pi_i \pi_i \sigma^2$$

where the field is $\phi = v + \sigma + \pi_{1,2,3}$. All of the quadratic $\pi_i \pi_i$ terms in Lagrangian vanish so the Lagrangian has one massive σ boson and three massless pions. The massless scalar particle is called the Goldstone bosons for any one spontaneously broken continuous symmetry. The Goldstone bosons number is equal to broken generator numbers. For example, there are $\frac{1}{2}N(N-1)$ generators when the Lagrangian considers an $O(N)$ continuous symmetry.

3.2 Spontaneous Breaking of Local Symmetries

In the local $U(1)$ gauge transformation situation, the Lagrangian density is invariant. It requires the massless gauge fields and electromagnetic field so the Lagrangian density becomes

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathcal{V}(\phi^\dagger \phi) \quad (3.2)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $D_\mu = \partial_\mu \phi - ieA_\mu \phi$ is the covariant derivative of ϕ . When the Lagrangian density is under the local gauge transformation, the local gauge transformation is

$$\phi_{new}(x) \rightarrow e^{ie\theta(x)} \phi(x)$$

$$A'_\mu(x) = A_\mu(x) + \partial_\mu \theta(x).$$

The minimum field energy is $|\phi| = v$ when the field A_μ vanishes, and the ϕ is constant. In the complex scalar field situation, the field form is set by

$$\phi' = v + \frac{h(x)}{\sqrt{2}}$$

where $h(x)$ is the real field so the Lagrangian density under the local symmetry breaking process becomes

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

$$\mathcal{L} = (D_\mu \phi')^\dagger (D^\mu \phi') - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2v^2} (\sqrt{2}vh(x) + \frac{h(x)^2}{2})^2 \quad (3.3)$$

where

$$\mathcal{L}_{free} = \frac{1}{2} \partial_\mu h \partial^\mu h - m^2 h^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + q^2 v^2 A_\mu A^\mu$$

and

$$\mathcal{L}_{int} = q^2 A_\mu A^\mu (\sqrt{2}vh + \frac{1}{2}h^2) - \frac{m^2 h^2}{2v^2} (\sqrt{2}vh + \frac{1}{4}h^2).$$

In \mathcal{L}_{free} form, the single scalar field $h(x)$ interprets the spinless bosons which mass is $\sqrt{2}m$, the vector field A_μ interprets three vector bosons which mass is $\sqrt{2}qv$. The particle corresponds to the single scalar field that is called Higgs boson.

The Higgs Mechanism

•Abelian Case: The Lagrangian of Higgs in abelian case can be easily written

$$\mathcal{L} = \frac{-1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^\dagger D^\mu\phi + \mathcal{V}(\phi)$$

where $V(\phi)$ is the scalar potential of the system, D_μ is the covariant derivative, with its action on ϕ given by $D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$. The potential density is:

$$\mathcal{V}(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

In abelian case, the field is set by

$$\phi = \frac{1}{\sqrt{2}}[v + \phi_1 + i\phi_2].$$

The Lagrangian of the Higgs boson in an abelian case keeps following the vacuum expectation value when the Lagrangian of the Higgs boson considers the $\mu^2 < 0$ situation.

$$\langle 0|\phi|0\rangle \equiv \phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{\frac{1}{2}} = \frac{v}{\sqrt{2}}$$

The Lagrangian is invariant under the local $U(1)$ symmetry transformation so the Lagrangian can be written

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu + ieA_\mu)\phi^\dagger(\partial^\mu - ieA^\mu)\phi - \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - v\lambda\phi_1^2 + \frac{1}{2}e^2v^2A_\mu A^\mu - evA_\mu\partial^\mu\phi_2.\end{aligned}$$

The Lagrangian of the Higgs mechanism in abelian case has a photon mass term, a scalar particle ϕ_1 term and a massless particle term ϕ_2 . The bilinear $evA^\nu A_\nu\phi_2$ term vanishes in the Lagrangian of the Higgs mechanism in abelian case. The photon has absorbed the Goldstone bosons and becomes massive. The photon longitudinal polarization is Goldstone in the photon polarization space. It is the Higgs mechanism that allows the generation of gauge boson mass value.[1]

Chapter 4

Higgs Particle in the SM

4.1 The SM before Electroweak Symmetry Breaking

The Lagrangian density has the strong terms and the electroweak terms. The theory of the electroweak is the Glashow-Weinberg-Salam electroweak theory which describes the electromagnetic interactions and the weak interactions between the quarks and the leptons and based on the $SU(2)_L \times U(1)_Y$ group, but the electroweak theory in the Higgs mechanism is the Yang-Mills theory which is the non-abelian symmetry group when the standard model combines the strong interaction terms with the unified electroweak interaction between quarks. The standard model has two kinds of fields such as the matter fields and the gauge fields.

•The matter field: The matter field, consist of the quarks and leptons which are spin- $\frac{1}{2}$ fermions. There are three generations of left and of right chiral quarks and leptons that can be written by

$$f_{L,R} = \frac{1}{2}(1 \mp \gamma_5)f$$

where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. The left chiral fermions are weak isodoublet when the right chiral fermions are weak singlets so isospin can be $\frac{1}{2}$ or 0. More precisely, we have $I_3^L = \pm\frac{1}{2}$ and $I_3^R = 0$. The leptonic degrees of freedom are:

$$\begin{aligned} L_1 &= (\nu_e, e^-)_L, & R_1 &= e_R^-; \\ L_2 &= (\nu_\mu, \mu^-)_L, & R_2 &= \mu_R^-; \\ L_3 &= (\nu_\tau, \tau^-)_L, & R_3 &= \tau_R^- \end{aligned}$$

On the other hand, the quark degrees of freedom are:

$$\begin{aligned} Q_1 &= (u, d)_L, & U_1 &= u_R, & D_1 &= d_R; \\ Q_2 &= (c, s)_L, & U_2 &= c_R, & D_2 &= s_R; \\ Q_3 &= (t, b)_L, & U_3 &= t_R, & D_3 &= b_R. \end{aligned}$$

The hypercharge of the fermion Y_f can be defined by the isospin I_3 and the electric charge Q_f relation.

$$Y_f = 2Q_f - 2I_3$$

Left-handed		Charge Q_f	Weak Isospin I_3	Weak Hypercharge Y_f
Leptons	ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	-1
	e, μ, τ	-1	$-\frac{1}{2}$	-1
Quarks	u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$
	d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$

Right-handed		Charge Q_f	Weak Isospin I_3	Weak Hypercharge Y_f
Leptons	ν_e, ν_μ, ν_τ			
	e, μ, τ	-1	0	-2
Quarks	u, c, t	$+\frac{2}{3}$	0	$+\frac{4}{3}$
	d, s, b	$-\frac{1}{3}$	0	$-\frac{2}{3}$

•The gauge field: The gauge field describes the bosons that mediate the interactions. The electroweak theory has one field B_μ and other three fields $W_\mu^{1,2,3}$. The B_μ field corresponds to the $U(1)_Y$ group Y . Other three fields $W_\mu^{1,2,3}$ corresponds to $SU(2)_L$ group. The Pauli matrix is marked by

$$T^i = \frac{1}{2}\tau^i, \quad i = 1, 2, 3$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The commutation relations between Y and T that can be written by

$$[T^i, T^j] = i\epsilon^{ijk}T_k, \quad [Y, Y] = 0$$

where ϵ^{ijk} is the anti-symmetry tensor. The strong interaction fields have octet gluon field G_μ^i which corresponds to the $SU(3)_C$ group, obtain relations that are

$$[T^i, T^j] = if^{ijk}T_k; \quad \text{Tr}[T^iT^j] = \frac{1}{2}\delta_{ij}$$

where f^{ijk} is the tensor of the $SU(3)_C$ group. The individual field is

$$G_{\mu\nu}^i = \partial_\mu G_\nu^i - \partial_\nu G_\mu^i + g_s f^{ijk} G_\mu^j G_\nu^k,$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_2 \epsilon^{ijk} W_\mu^j W_\nu^k, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

where g_s is the coupling constant of the $SU(3)_C$, g_1 is the coupling constant of the $U(1)_Y$, and g_2 is the coupling constant of the $SU(2)_L$. The Lagrangian of the standard model can be written

$$\mathcal{L}_{SM} = -\frac{1}{4}G_{\mu\nu}^i G_i^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^j W_j^{\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} + L_i^\dagger iD_\mu \gamma^\mu L_i + e_{R_i}^\dagger iD_\mu \gamma^\mu e_{R_i} +$$

$$Q_i^\dagger iD_\mu \gamma^\mu Q_i + u_{R_i}^\dagger iD_\mu \gamma^\mu u_{R_i} + d_{R_i}^\dagger iD_\mu \gamma^\mu d_{R_i}$$

when the SM Lagrangian doesn't consider the mass terms. The D_μ is the covariant derivative. If the Lagrangian of the standard model adds the gauge boson mass terms, the Lagrangian will violate local gauge invariance. If the SM Lagrangian adds the fermion mass terms, the Lagrangian will be manifestly non-invariant under the isospin symmetry transformations when e_L is a member of an $SU(2)_L$ doublet when e_R is a member of a singlet. It has two kinds of method to solution. First strategy is the Higgs-Brout-Englert-Guralnik-Hagen-Kibble mechanism of spontaneous symmetry breaking. Other strategy is the Higgs mechanism for short.

4.2 The Higgs Mechanism in the SM

The Higgs mechanism has two cases to discuss such as an abelian case and an non-abelian case. The Higgs mechanism of the SM is in non-abelian space.

•Non-Abelian Case: The non-abelian case of the SM considers more particles in the Higgs mechanism such as the W bosons, the Z bosons and the photon. The field chooses a complex $SU(2)$ doublet of scalar field ϕ in the Lagrangian of Higgs mechanism of non-abelian case.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad Y_\phi = 1$$

The Lagrangian in SM of the non-abelian case ignores the strong interaction parts so the Lagrangian can be written

$$\mathcal{L}_{SM} = \mathcal{L}_{Bosons} + \mathcal{L}_{Fermions} + \mathcal{L}_{WeakInteraction}$$

The Lagrangian of SM adds the invariant terms of the scalar field \mathcal{L}_S so the Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_S$$

where \mathcal{L}_S is defined by

$$\mathcal{L}_S = (D^\mu \phi)^\dagger (D_\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

The non-abelian case follows the vacuum expectation value when the mass $\mu^2 < 0$ case.

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

The vacuum value is given by

$$v = \left(-\frac{\mu^2}{\lambda} \right)^{\frac{1}{2}}$$

The field form in the standard model can be written

$$\phi(x) = \begin{pmatrix} \phi_2 + i\phi_1 \\ \frac{1}{\sqrt{2}}(v + H) - i\phi_3 \end{pmatrix} = \frac{1}{\sqrt{2}} e^{-i\theta_i(x)\tau^i(x)/v} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

where H is the Higgs field, $\phi_{1,2,3}$ is the real scalar field. The field function does a gauge transformation so the field function becomes

$$\phi \rightarrow e^{i\theta_i(x)\tau^i(x)/v} \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

The Lagrangian in SM of the Higgs mechanism only considers the kinetic part so the kinetic term $|D_\mu\phi|^2$ of the Lagrangian can be written

$$\begin{aligned} |D_\mu\phi|^2 &= \left| \left(\partial_\mu - ig_2 \frac{\tau_i}{2} W_\mu^i - ig_1 \frac{B_\mu}{2} \right) \phi \right|^2 \\ &= \frac{1}{2} \left| \begin{pmatrix} \partial_\mu - \frac{i}{2}(g_2 W_\mu^3 + g_1 B_\mu) & -\frac{ig_2}{2}(W_\mu^1 - iW_\mu^2) \\ -\frac{ig_2}{2}(W_\mu^1 + iW_\mu^2) & \partial_\mu + \frac{i}{2}(g_2 W_\mu^3 - g_1 B_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \right|^2 \\ &= \frac{1}{2} (\partial_\mu H)^2 + \frac{1}{8} g_2^2 (v + H)^2 |W_\mu^1 + iW_\mu^2|^2 + \frac{1}{8} (v + H)^2 |g_2 W_\mu^3 - g_1 B_\mu|^2 \end{aligned}$$

The new field of W_μ^\pm and of Z_μ is defined by the kinetic term of the Lagrangian of the extra scalar field when A_μ is the orthogonal fields to Z_μ .

$$W^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad Z_\mu = \frac{g_2 W_\mu^3 - g_1 B_\mu}{\sqrt{g_1^2 + g_2^2}}, \quad A_\mu = \frac{g_2 W_\mu^3 + g_1 B_\mu}{\sqrt{g_1^2 + g_2^2}}$$

The W bosons and the Z boson will have mass when the photon is still massless so the W boson and the Z boson can be defined by

$$M_W = \frac{v}{2} g_2, \quad M_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2}, \quad M_A = 0$$

Three gauge Goldstone bosons since the Lagrangian are absorbed by the W^\pm bosons and Z boson from their longitudinal components when the spontaneously breaking symmetry $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$. The fermion part is also use same way so the Yukawa Lagrangian of the $SU(2)_L \times U(1)_Y$ invariant can be written

$$\mathcal{L}_F = -\lambda_e L^\dagger \phi e_R - \lambda_d Q^\dagger \phi d_R - \lambda_\mu Q^\dagger i\tau_2 \phi^\dagger \phi u_R + \dots$$

The Fermion mass can be defined by \mathcal{L}_F .

$$m_e = \frac{\lambda_e v}{\sqrt{2}}, \quad m_\mu = \frac{\lambda_\mu v}{\sqrt{2}}, \quad m_d = \frac{\lambda_d v}{\sqrt{2}}$$

If the Lagrangian only considers the Higgs fields when the Higgs particle in the standard model, the Lagrangian will be written

$$\mathcal{L}_H = \frac{1}{2} (\partial_\mu H)(\partial_\mu H) - \mathcal{V}$$

where the scalar potential of system is $\mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$. The scalar potential of system is

$$\mathcal{V} = -\frac{1}{2} \lambda v^2 (v + H)^2 + \frac{\lambda}{4} (v + H)^4$$

when

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad \text{and} \quad v^2 = -\frac{\mu^2}{\lambda}$$

that is vacuum value.

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

The Higgs bosons mass can easy and simply read

$$M_H^2 = 2\lambda v^2 = -\mu^2$$

from the Lagrangian of Higgs field. The Higgs self-interactions are defined by the Feynman rules so the Higgs self-interaction forms can be written

$$g_{H^3} = 3!\lambda v, \quad g_{H^4} = 4!\frac{\lambda}{4} = 3\frac{M_H^2}{v^2}$$

If the system considers the Higgs bosons couples to the fermions and the bosons, the Lagrangian will be described by

$$\mathcal{L}_{Bosons} \sim M_V^2 \left(1 + \frac{H}{v}\right)^2, \quad \mathcal{L}_{Fermions} \sim -m_f \left(1 + \frac{H}{v}\right).$$

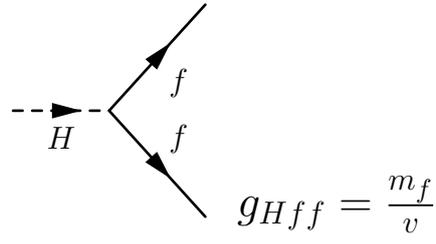
The coupling terms also can be defined by the Lagrangian of Higgs field when the Higgs bosons couples to the fermions and the bosons so the coupling terms can be written

$$g_{Hff} = \frac{m_f}{v}, \quad g_{HVV} = 2\frac{M_V^2}{v}$$

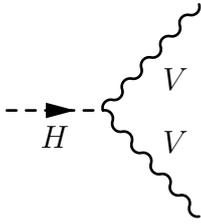
where V means the bosons, f means the fermions. The vacuum expectation value can be defined by the W boson mass or the Fermi constant G_F that is determined by the muon decay.

$$v = \frac{g_2}{2M_W} \quad \text{or} \quad v = (\sqrt{2}G_F)^{-\frac{1}{2}} \approx 246 \quad GeV$$

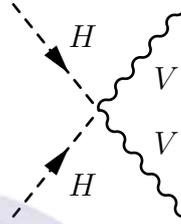
The general coupling forms can be defined by the feynman rule so the Higgs bosons couples to the fermions and the bosons that can easy and simply be written[2]



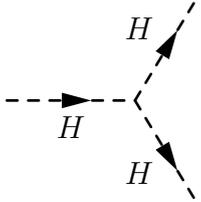
$$g_{Hff} = \frac{m_f}{v}$$



$$g_{HVV} = \frac{2m_V^2}{v}$$



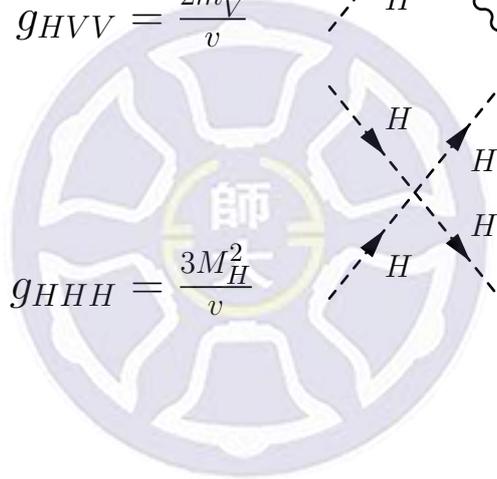
$$g_{HHVV} = \frac{2m_V^2}{v^2}$$



$$g_{HHH} = \frac{3M_H^2}{v}$$



$$g_{HHHH} = \frac{3M_H^2}{v^2}$$



Chapter 5

Low Energy Theorem

The low-energy theorem for Higgs bosons considers the interaction term of the Lagrangian in the standard electroweak theory so the Lagrangian of the interaction part can be written

$$\mathcal{L}_{Int} = -(1 + \frac{H}{v}) \sum_f m_f f f^\dagger - (1 + \frac{H}{v})^2 (m_W^2 W^{\mu+} W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z^\mu)$$

where f means the fermion fields, W means the W boson fields, Z means the Z boson field. The vacuum expectation value is 246 GeV when the Fermi constant is determined by the muon decay.

$$v \equiv (\sqrt{2} G_F)^{-\frac{1}{2}} \simeq 246 \text{ GeV}$$

where $G_F = 1.16 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant. The Lagrangian of the interaction term considers a Higgs field with zero four-momentum that implies the Higgs field is a constant field, all of mass parameters can be redefined by

$$m_i \rightarrow m_i (1 + \frac{H}{v}).$$

When the Higgs momentum approaches zero, the amplitude form can be written

$$\lim_{P_H \rightarrow 0} \mathcal{M}(A \rightarrow B + H) = \frac{1}{v} (\sum_f m_f \frac{\partial}{\partial m_f} + \sum_V m_V \frac{\partial}{\partial m_V}) \mathcal{M}(A \rightarrow B)$$

where f sums over the fermions, V sums over the bosons. Higgs production by gluon fusion in SM only considers the fermion loops. Two gamma decay width of the Higgs boson in SM considers the fermion loops and the boson loops.

This paper only considers the case of Higgs production by gluon-gluon fusion so it only has two kind of cases such as the light Higgs case ($2m_f > m_H$) and the heavy Higgs case ($2m_f < m_H$).

This paper focuses on the light Higgs case because the Higgs boson is discovered at 125 GeV by the CERN, and we assume that new heavy particles are heavy than 125 GeV. The Lagrangian of the Higgs production by gluon-gluon fusion with the gluon kinetic energy can be written

$$\mathcal{L} = \frac{-1}{2g_s^2}(\partial_\mu G_\nu^i - \partial_\nu G_\mu^i - f_{ijk}G_\mu^j G_\nu^k)^2(1 - \frac{N_H\alpha_s}{3\pi v}H)$$

where N_H is the number of quark flavor. When the Higgs field is a constant field, the Higgs field H interactions can be produced by rescaling α_s so the α_s term can be written as

$$\alpha_s \rightarrow \alpha_s + \delta\alpha_s$$

where

$$\delta\alpha_s = \frac{N_H\alpha_s^2}{3\pi v}H.$$

The effective Lagrangian can be written

$$\begin{aligned}\mathcal{L}_{Hgg} &\rightarrow \frac{N_H}{12\pi v}(\alpha_s + \delta\alpha_s)HG_{\mu\nu}^i G_{\mu\nu}^i \\ \mathcal{L}_{Hgg} &\rightarrow \frac{N_H}{12\pi v}(\alpha_s + \frac{N_H\alpha_s^2}{3\pi v}H)HG_{\mu\nu}^i G_{\mu\nu}^i.\end{aligned}$$

When the Lagrangian considers the light Higgs case, the Lagrangian of interaction part also follows

$$\lim_{P_H \rightarrow 0} \mathcal{M}(A \rightarrow B + H) = \frac{1}{v}(\sum_f m_f \frac{\partial}{\partial m_f} + \sum_V m_V \frac{\partial}{\partial m_V})\mathcal{M}(A \rightarrow B)$$

where f sums over the fermion, V is W bosons and Z bosons. [3][4]

Chapter 6

Phenomenology

In the Standard Model (SM), the dominant Higgs production channel is via gluon fusion, the relevant formulae of which will be reviewed in the next subsection. According to the low-energy theorem for Higgs bosons, when new particles are introduced, the SM cross section (Eq. (6.1) below) will receive additional contributions in which the top quark loops are replaced by new particle loops. The resulting formulae will depend on the new particle masses and additional coupling parameters. In what follows we will consider the Higgs production by gluon-gluon fusions from the proton-proton collider experiments. The MSTW2008lo68cl database will be used, and the HIGLU program in Fortran language will be adopted.

6.1 Higgs Production by Gluon-Gluon Fusion in SM

In the standard model, the high probability of the Higgs boson in the gluon-gluon fusion mechanism is mediated by the quarks. In the Higgs production, the gluon-gluon fusion mechanism only considers the top quark because the coupling form and the event rate are proportional.

$$g_{Hff} = \frac{m_f}{v}, \quad g_{HVV} = \frac{2m_V^2}{v}, \quad g_{HHH} = \frac{3M_H^2}{v}$$

The leading order cross section in Higgs production can be expressed by two gluon decay of the Higgs boson. It can be written as

$$\hat{\sigma}_{LO}(gg \rightarrow H) = \Gamma_0^H M_H^2 \delta(\hat{s} - M_H^2)$$

where Γ_0^H is two gluon decay of the Higgs boson cross section, \hat{s} is the gg invariant energy. The leading order approximation uses the Breit-Wigner form of the Higgs boson. When the δ distribution width is non-zero, the $\delta(\hat{s} - M_H^2)$ should be replaced via the following:

$$\delta(\hat{s} - M_H^2) \rightarrow \frac{1}{\pi} \frac{\hat{s}\Gamma_H/M_H}{(\hat{s} - M_H^2)^2 + (\hat{s}\Gamma_H/M_H)^2}$$

where Γ_H is the lowest-order two gluon decay width of the Higgs boson. The cross section of the lowest-order two gluon decay of the Higgs boson is

$$\Gamma_0^H = \frac{G_F \alpha_S^2(\mu_R^2)}{288\sqrt{2}\pi} \left| \frac{3}{4} \sum_q A_{\frac{1}{2}}(\tau_q) \right|^2 \quad (6.1)$$

where μ_R is the renormalization scale. In the proton-proton case, the cross section at leading order allows the narrow-width approximation that can be read

$$\sigma_{LO}(pp \rightarrow H) = \Gamma_0^H \tau_H \frac{d\mathcal{L}_{gg}}{d\tau_H}$$

where $\tau_H = M_H^2/s$ with s being the total collider scale. The $d\mathcal{L}_{gg}/d\tau_H$ term can be defined by

$$\frac{d\mathcal{L}_{gg}}{d\tau_H} = \int_{\tau}^1 \frac{dx}{x} g(x, \mu_F^2) g\left(\frac{\tau_H}{x}, \mu_F^2\right)$$

where μ_F is the factorization scale.[2]

6.2 Low Energy Theorem in New Physics

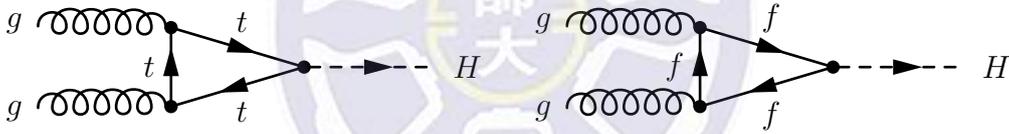
This section will consider the extra particles in the Feynman diagram of Higgs production by gluon fusion so there are three parts such as fermion case, scalar case and contribution part. The experiment mechanism chooses proton-proton experiment at 14 TeV when Higgs boson is at $m_H = 125$ GeV and top quark is at $m_t = 175$ GeV.

Fermion Case

The cross section of SM adds new cross section in fermion case. Top quark loops are replaced by new fermion particle loop. [5] Lagrangian of SM includes extra term which is:

$$\mathcal{L} \supset m_{f_0} \bar{f}f + \frac{c_f}{\Lambda} \phi^\dagger \phi \bar{f}f, \quad m_f = m_{f_0} \left(1 + \frac{c_f v^2}{2m_{f_0} \Lambda}\right)$$

where ϕ is ϕ of Higgs boson, f is extra new fermion field.



The cross section of SM and the new cross section consider at leading order term. If the coupling strength is very large, the discoverable events will also be very more. If particle mass is very large, the coupling variable will also be very large in SM case.

We can defined a new parameter R_f which describes new physics phenomenology. R_f is defined by the new cross section over the SM cross section in SM. It can be written

$$R_f = \frac{\sigma_{new}}{\sigma_{SM}} = \left| 1 + c_f \frac{v^2}{2\Lambda m_f} \frac{A_{\frac{1}{2}}(\tau_f)}{A_{\frac{1}{2}}(\tau_t)} \right|^2$$

where m_f is the new particle mass, Λ is the adjustable variable and describes the new particle mass threshold when the new particle couples to the Higgs bosons, and c_f is coupling variable. We see that $R_f > 0$ by definition.

$A(\tau)$ function can be defined by low-energy theorem for Higgs bosons. It can be written

$$A_{\frac{1}{2}}(\tau) = 2\tau[1 + (1 - \tau)f(\tau^{-1})]$$

where $f(\tau) = \arcsin^2(\sqrt{\tau})$, $\tau_i = 4m_i^2/m_H^2$. The R_f will be equal to 1 when the new particle mass is infinite, the coupling variables are fixed $c_f = \pm 1$, and Λ sets 1000 GeV in Fig 6.1.

$$\lim_{m_f \rightarrow \infty} R_f = \lim_{m_f \rightarrow \infty} \frac{\sigma_{new}}{\sigma_{SM}} = 1$$

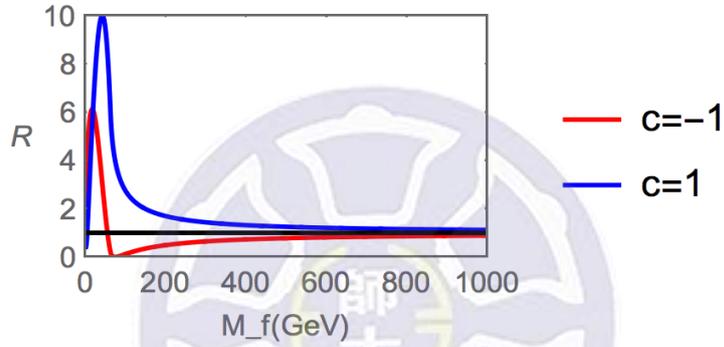


Figure 6.1: R_f is close to 1 when m_f is infinite. Blue line is $c = 1$. Red line is $c = -1$. Black line is Higgs cross section in SM.

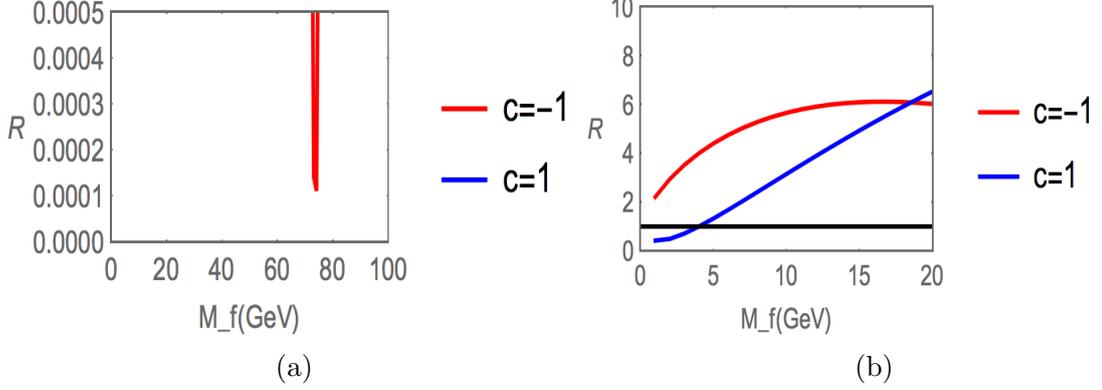


Figure 6.2: Parts of Fig 1 shows is different scales. The left figure focuses on the low point of the red line when $c = -1$. The right figure focuses on the minimum range of the mass from 1 GeV to 20 GeV when the extra particle mass is infinite.

When the coupling variable is $c_f = -1$, the R_f of the red line has a minimum that is at $m_f = 76$ GeV in Fig. 6.2(a) because destructive interference effect. The R_f is when extra particle mass is from 0 to 20 GeV, c_f is fixed in Fig. 6.2(b). When c_f is from -2 to 2, m_f is from 1 GeV to 1 TeV, R_f projects on $c_f - m_f$ plane and is calculated by the HIGLU code in Fig. 6.3. The average value of experiment will now be at $R_f = 1.03$. The CMS experiment discovers that new particles are when the coupling variable is negative. The ATLAS experiment discovers that new particles are when the coupling variable is positive.

$$R_{ggF} = \begin{cases} 1.25_{-0.21}^{+0.24}, & (\text{ATLAS}) \\ 0.84_{-0.16}^{+0.19}, & (\text{CMS}) \end{cases} \quad (6.2)$$

The combined result for the two collaborations are then:[8]

$$R_{ggF} = 1.03_{-0.15}^{+0.17} \text{ (ATLAS+CMS)}$$

The average value of experiment will now be positive when the coupling variable is positive. When new particle mass m_f increases with increasing coupling variable c_f , the coupling variable is slowly increasing in Fig. 6.3. If we want to conform experimental average value when coupling variable $c_f = 1$, the new fermion particle has two kind of mass ranges: 1.) $m_f \approx 10$ GeV. 2.) $m_f > 620$ GeV. If we want to conform experimental average value when coupling variable $c_f = -1$, the new fermion particle mass has two kind

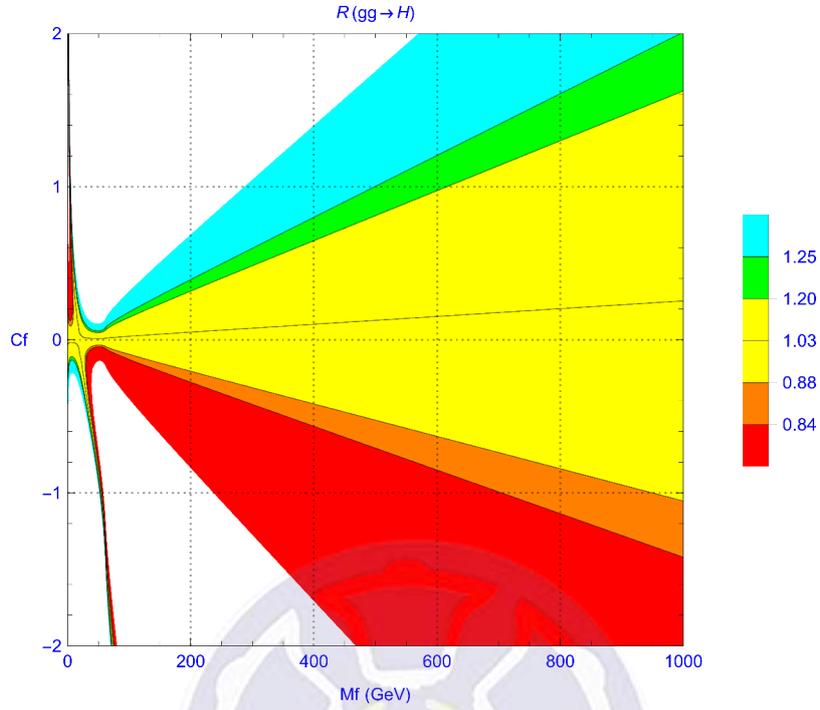


Figure 6.3: R_f projects on c_f - m_f plane when c_f is from -2 to 2, m_f is from 1 GeV to 1 TeV. Fig. 6.3 is made by Higlū program. The Λ sets 1000 GeV.

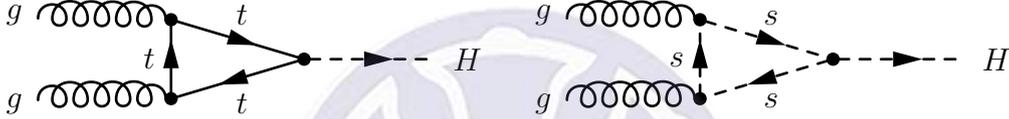
of mass range: 1.) $m_f \approx 50$ GeV. 2.) $m_f > 950$ GeV. In short, Fermion case can apply in NP such as little Higgs theorem.

Color Scalar Case

The cross section of Higgs production by gluon fusions adds the new cross section in SM. Top quark loops is replaced by new color scalar loops. The scalar particles has two kind of cases such as complex scalar case and real scalar cases.[6][7]Lagrangian of SM includes extra term which is:

$$\mathcal{L} \supset -m_{s_0}^2 S^\dagger S - c_S \phi^\dagger \phi S^\dagger S, \quad m_s^2 = m_{s_0}^2 \left(1 + c_S \frac{v^2}{2m_{s_0}^2}\right)$$

where S is extra scalar particle field, c_S is coupling variable, and m_s is scalar particle.



From the Feynman diagrams, the SM cross section adds the new cross section. In new physics, the cross section will now be

$$\sigma_S = \frac{G_F \alpha_S^2}{288 \sqrt{2} \pi} |A_{\frac{1}{2}}(\tau) + K b C(r) c_S \frac{v^2}{2m_s^2} A_0(\tau)|^2$$

where $C(r)$ is the Casimir invariant of the $SU(3)_C$, $K = 3$ in the case of weak triplet, $b = N_s/2$. N_s is the degree of freedom of the scalar fields, hence is equal to 1 and 1 for the complex and real case, respectively. $A_0(\tau)$ function is defined by

$$A_0(\tau) = -[\tau - \tau^2 f(\tau^{-1})]$$

where $\tau = 4m_i^2/m_H^2$ and $f(\tau^{-1}) = \arcsin^2(\sqrt{\tau^{-1}})$. We can define a new parameter R_S which describes new physics phenomenology in scalar case. R_S is defined by the new cross section over the SM cross section in SM. It can be written

$$R_S = \left|1 + 4 \cdot b c_S \frac{v^2}{2m_s^2} \frac{A_0(\tau)}{A_{\frac{1}{2}}(\tau)}\right|^2 \quad (6.3)$$

Real Case

If the scalar particle is the real scalar particle, the b parameter of the cross section will be set $1/2$. Top quark loops are replaced by real scalar loops. The R_S becomes

$$R_S = \left| 1 + 2c_S \frac{v^2}{m_s^2} \frac{A_0(\tau)}{A_{\frac{1}{2}}(\tau)} \right|^2 > 0.$$

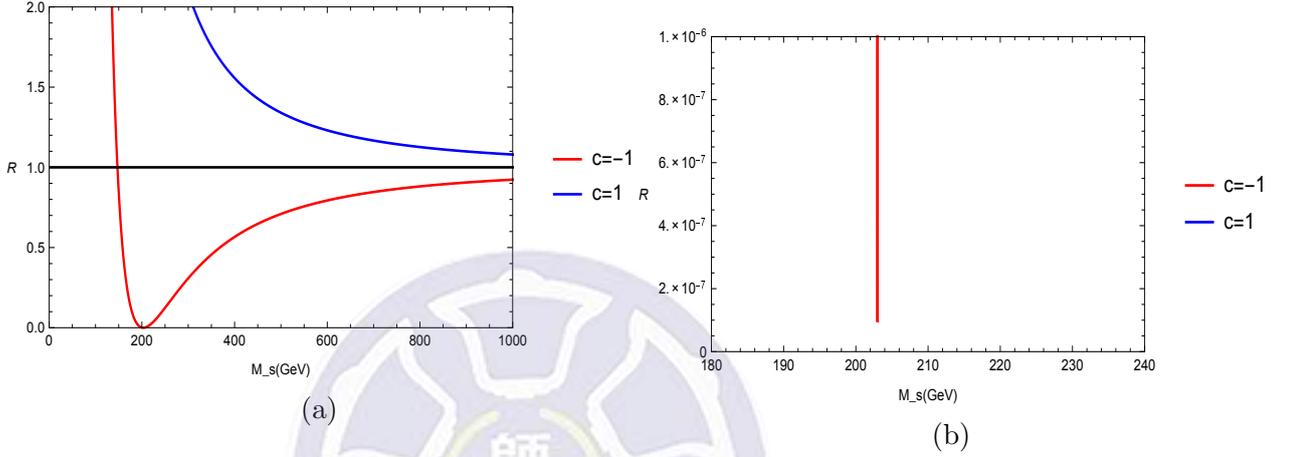
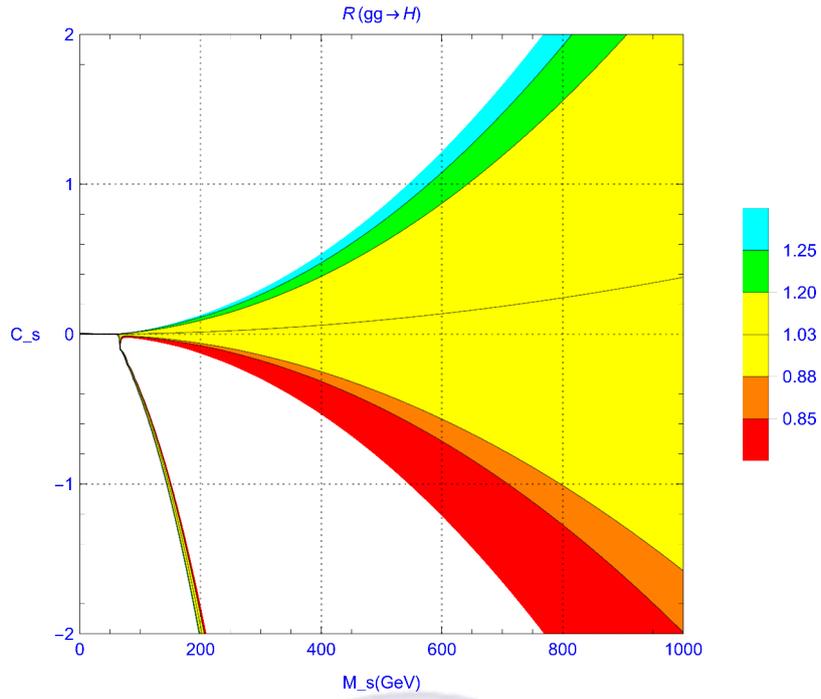


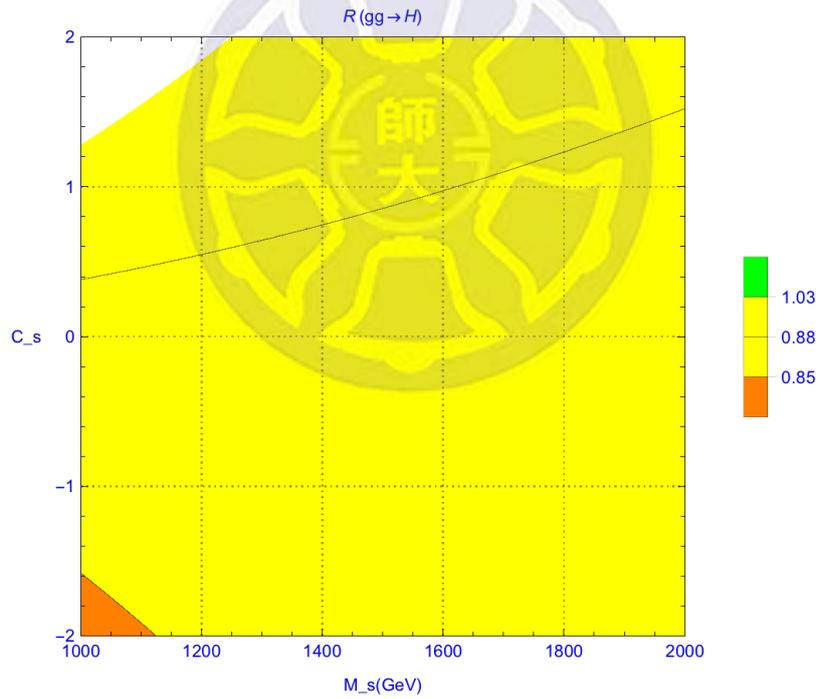
Figure 6.4: R_S is close 1 when m_s is infinite. R_S has a minimum when $c_S = 1$. R_S has not a minimum when $c_S = -1$. The minimum point is at $m_s = 203$ GeV when $c_S = 1$. It is made by gluon program in Fortran.

When the coupling variable is fixed, the extra particle mass is infinite, the R_S will be equal to 1 in Fig. 6.4(a). We can find out Fig. 6.4(a) has two kinds of phenomenology such as destructive interference and constructive interference. Destructive interference is when coupling variable $c_S = -1$. Constructive interference is when coupling variable $c_S = 1$. When the coupling variable is equal to -1 , R_S has a minimum that is at $m_s = 203$ GeV in Fig. 6.4(b), but this minimum is produced by destructive interference effect. The R_S is increased when the extra particle mass is close to 0.

R_S projects on $c_S - m_s$ plane when c_S is from -2 to 2 , m_s is from 1 GeV to 1 TeV in Fig. 6.5(a). R_S projects on $c_S - m_s$ plane when c_S is from -2 to 2 , m_s is from 1 TeV to 2 TeV in Fig. 6.5(b). According to Fig. 6.5, we can find out allowable range is very big. If we want to conform experimental average value, the extra particle must exceed 1.6 TeV in Fig. 6.5(b) when $c_S = 1$. If coupling variable $c_S = -1$, extra scalar particle mass has two kinds of range: 1.) $m_s \approx 147$ GeV 2.) $m_s \approx 800$ GeV.



(a) The R_S projects on c_S - m_S plane when c_S is from 2 to -2 . It is made by Higlū program.



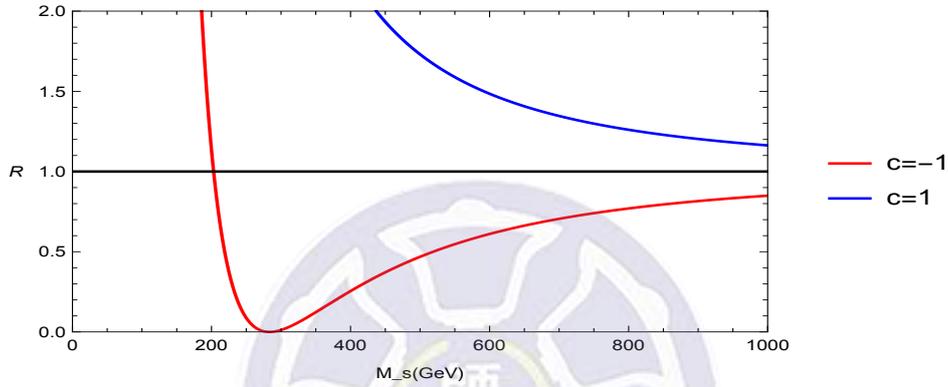
(b) R_S projects on c_S - m_S plane when the extra particle mass is from 1 GeV to 3 TeV.

Figure 6.5

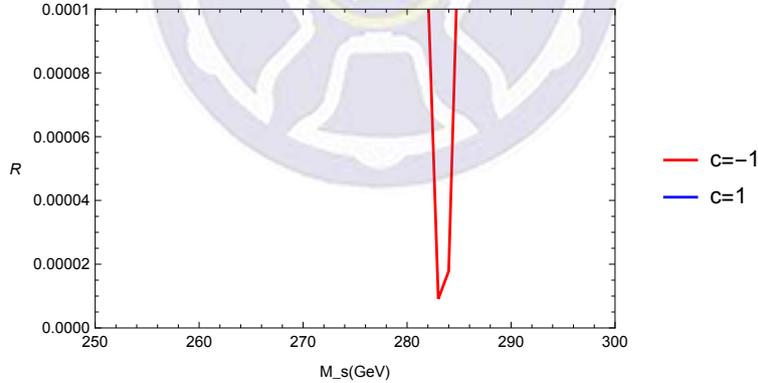
Complex Scalar Case

Real scalar loops are replaced by the complex scalar loops. The b parameter sets 1 in complex scalar case because degrees of freedom of complex scalar particle has two. R_S will become:

$$R_S = \left| 1 + 4c_S \frac{v^2}{m_s^2} \frac{A_0(\tau)}{A_{\frac{1}{2}}(\tau)} \right|^2, \quad R_S > 0.$$



(a) R_S is close 1 when $m_s = \infty$. The blue line is when the coupling variable is 1. The red line is when the coupling variable is -1.



(b) R has a minimum which is at 283 GeV when $c_s = 1$.

Figure 6.6

When the coupling variable is fixed, $m_s = \infty$, the R_S will be close to 1 in Fig. 6.6(a). We can find out two line such as destructive interference (red line) and constructive interference (blue line) in Fig. 6.6(a). Destructive interference is when $c_S = -1$ and has a minimum which is at 283 GeV and produced by destructive interference effect in Fig. 6.6(b). Constructive interference is

when $c_S = 1$ and hasn't the minimum in Fig. 6.6(a)

R_S projects on $c_S - m_s$ plane when c_S is from -2 to 2 , m_s is from 1 GeV to 1 TeV in Fig. 6.7. When we change the x-axis of the extra particle mass range that is from 1 GeV to 3 TeV, c_s is from -2 to 2 , R_S projects on $c_S - m_s$ plane in Fig. 6.8.

According to Fig. 6.7, we can find out allowable range becomes small when extra particle is complex particle. Extra particle candidate condition has an upward trend. If we want to conform experimental value when coupling variable $c_S = 1$, the extra complex scalar mass exceeds 2.2TeV ($m_s > 2.2$ TeV). If coupling variable $c_S = -1$, the extra complex scalar particle has two kind of mass range: 1.) $m_s \approx 200$ GeV. 2.) $m_s > 1.1$ TeV. In short, color scalar case can apply in SUSY, dark matter candidates.



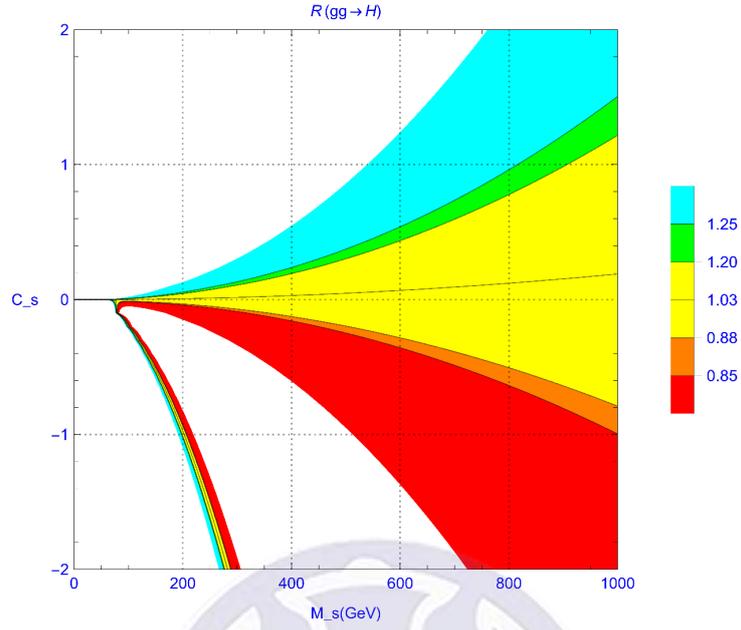


Figure 6.7: R_S projects on c_S - m_s plane when c_S is from 2 to -2 . It is made by highlu program.

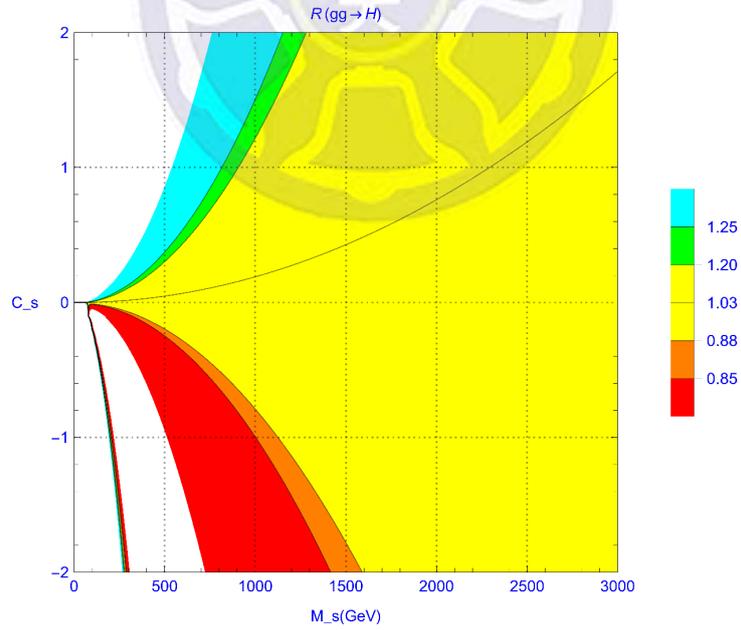


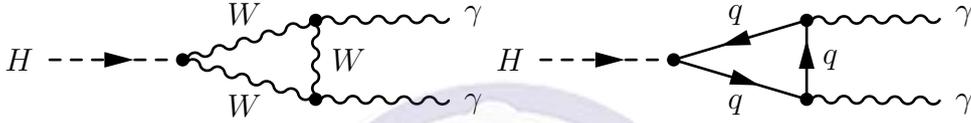
Figure 6.8: R_S projects on c_S - m_s plane when extra particle mass is from 1 GeV to 3 TeV, c_S is from -2 to 2.

Contribution to Two Gamma Decay of the Higgs Boson

All of case will contribute to two gamma decay of the Higgs boson. Two gamma decay of the Higgs boson only considers the quark loops.[3][5] Lagrangian of SM includes extra Lagrangian term which is:

$$\mathcal{L} \supset m_{q_0} \bar{q}q + \frac{c_q}{\Lambda} \phi^\dagger \phi \bar{q}q, \quad m_q = m_{q_0} \left(1 + \frac{c_q}{m_{q_0} \Lambda}\right)$$

where q is new particle filed, c_q is coupling variable.



Two gamma decay width of the Higgs boson of SM adds the new decay width. W loops and top quark loops in SM are replaced by new quark loops. It can also define new parameter R_Γ which describes new physics phenomenology. The R_Γ is defined by the new decay width over the SM decay width. R_Γ can be written:

$$R_\Gamma = \frac{\Gamma_{new}}{\Gamma_{SM}} = \left| 1 + c_q \frac{v^2}{m_q \Lambda} \frac{A_{\frac{1}{2}}(\tau)}{A_1(\tau) + A_{\frac{1}{2}}(\tau)} \right|^2$$

where A_1 is defined by

$$A_1(\tau) = -[1 + 3\tau + 3(2\tau - \tau^2)f(\tau^{-1})].$$

The R_Γ will be close to 1 when the extra particle mass m_s is infinite in Fig. 6.9(a). The R_Γ won't be zero when c_q is 1 or -1 in Fig. 6.9(b).

$$\lim_{m_l \rightarrow \infty} R_\Gamma = \lim_{m_l \rightarrow \infty} \frac{\Gamma_{new}}{\Gamma_{SM}} = 1$$

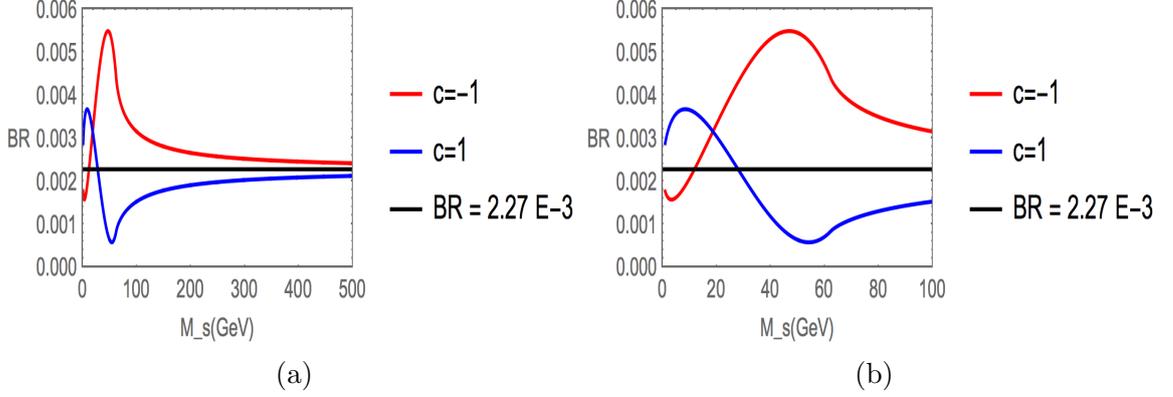


Figure 6.9: It is made by Hdecay program when the coupling variable is fixed. The left part interprets R_Γ is close branching ratio of SM when $m_s = \infty$. The right part interprets R_S is when the mass range is from 1 GeV to 100 GeV. The black line is the SM branching ratio (BR).

R_Γ has two phenomenology in Fig. 6.9(a) such as destructive interference and constructive interference. Destructive interference is when coupling variable $c_q = 1$ and has a minimum which is at 55 GeV in Fig. 6.9(b) and produced by destructive interference effect. Constructive interference is when coupling variable $c_q = -1$ and has a maximum which is at 45 GeV and produced by constructive interference effect in Fig. 6.9(b).

R_Γ is calculated by HEDCAY program and projects on $c_q - m_q$ plane when c_q is from -2 to 2, m_q is from 1 GeV to 1 TeV in Fig. 6.10. R_Γ projects on $c_q - m_q$ plane when c_q is from -2 to 2, m_q is from 1 TeV to 2 TeV in Fig. 6.11.

The average value of experiment will be at[8]

$$R_{F\gamma\gamma} = \begin{cases} 1.15^{+0.27}_{-0.25}, & \text{ATLAS} \\ 1.12^{+0.25}_{-0.23}, & \text{CMS} \end{cases} \quad (6.4)$$

The combined result for the two collaborations are then:

$$R_{F\gamma\gamma} = 1.16^{+0.20}_{-0.18}.$$

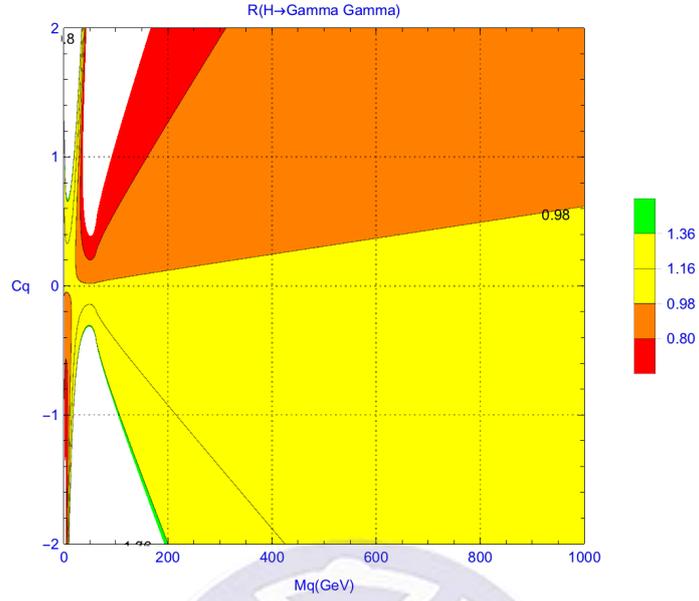


Figure 6.10: R_Γ projects on $c_q - m_q$ plane when exotic mass is from 1 GeV to 2 TeV, c_q is from -2 to 2. The Λ sets 1000 GeV.

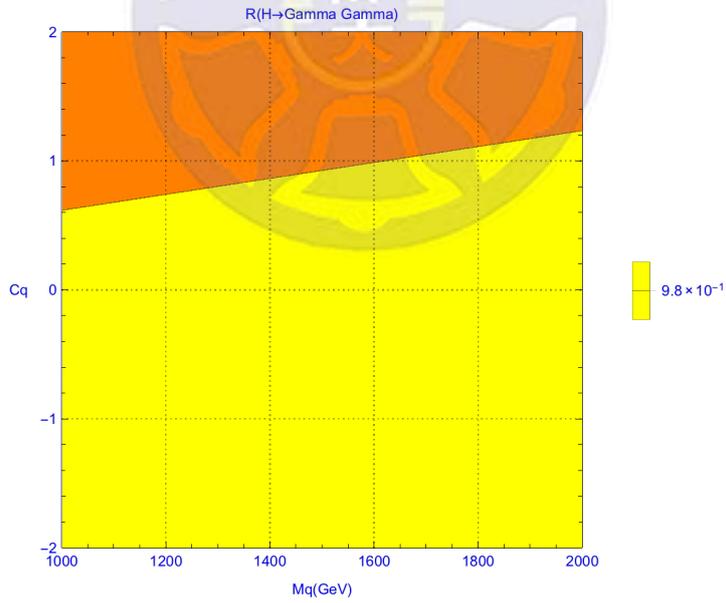


Figure 6.11: R_Γ projects on $c_q - m_q$ plane when exotic mass is from 1 TeV to 2 TeV, c_q is from -2 to 2.

If we want to conform experimental average value when coupling variable $c_q = -1$, exotic particle mass has two kinds of mass range: 1.) $m_q \approx 15$ GeV. 2.) $m_q > 220$ GeV. If we want to conform experimental value when coupling variable $c_q = 1$, exotic particle has two kind of mass range.: 1.) $m_q \approx 20$ GeV. 2.) $m_q > 1.6$ TeV. In short, this case also can apply in little Higgs theorem.



Chapter 7

Conclusion

In SM, we know top quark of Higgs production by gluon fusions is the major effect because coupling variable is very big so we only consider the top quark effect when exotic particle joins in Higgs production by gluon fusion. According to low-energy theorem for Higgs boson, we can calculate when top quark loop of Higgs productions by gluon fusion replaced by exotic particle loops so it will have three cases such as fermion particle case, color scalar case and contribution case.

In Fermion case, R_f has two kind of phenomenology. For example, destructive interference is when $c_f = -1$. Constructive interference is when $c_f = 1$. Experimental average value is now at 1.03. If we want to conform to experimental average value, exotic particle mass will have two kind of mass range: 1.) Destructive interference: $m_f \approx 50$ GeV or $m_f > 950$ GeV. 2.) Constructive interference: $m_f \approx 10$ GeV or $m_f > 620$ GeV. The fermion case can apply in little Higgs theorem.

In color scalar case, R_S has two kind of cases such as complex scalar case and real scalar case. It also has interference phenomenology. For example, destructive interference is when $c_S = -1$. Constructive interference is when $c_S = 1$. If we want to conform to experimental value, exotic particle has two kinds of mass range when $c_S = -1$. Real case: $m_s \approx 150$ GeV or $m_s > 800$ GeV. Complex case: $m_s \approx 200$ GeV or $m_s > 1.1$ TeV. If we want to conform to experimental value, exotic particle is in height mass ranges when $c_S = 1$. Real case: $m_s > 1.1$ TeV. Complex case: $m_s > 2.2$ TeV. The color scalar case can apply in SUSY.

Fermion case and color scalar case will contribute to two gamma decay of Higgs boson. We consider W boson loop and top quark loop of two gamma

decay of Higgs boson is replaced by exotic quark loops. R_Γ has two kinds of phenomenology such as destructive interference and constructive interference. Destructive interference is when $c_q = 1$. Constructive interference is when $c_q = -1$. If we want to conform to experimental value, exotic particle has two kinds of mass range: Destructive interference: $m_q \approx 15$ GeV or $m_q \approx 220$ GeV. Constructive interference: $m_q \approx 20$ GeV or $m_q > 1.6$ TeV.

In short, we can know that if we consider exotic particle adds in Higgs production by gluon fusions, exotic particle is in height mass ranges or low mass range when Higgs boson is at 125 GeV. In color scalar case, we can know allowable range becomes small when exotic particle is complex particle. Fermion case and contribution case can apply in little Higgs theorem. Color scalar case can apply in SUSY. All of case in Higgs production will contribute to two gamma decay of the Higgs boson. In two gamma decay of the Higgs boson, we can discovery the allowable range which is very big.



Chapter 8

Biography



Bibliography

- [1] **Introduction to the Standard Model of Particle Physics**, by W. N. Cottingham and D. A. Greenwood (ISBN 978-0-521-85249-4)
- [2] Djouadi, Abdelhak:**The Anatomy of Electro-Weak Symmetry Breaking Tome I: The Higgs boson in the Standard Model**, by Abdelhak Djouadi, arXiv:0503172v2
- [3] Mikhail A. Shifman, A.I. Vainshtein, M.B. Voloshin, Valentin I. Zakharov:**Low Energy Theorems for Higgs Boson Coupling to Photons**, Sov. J. Nucl. Phys. 30, 711(1979)
- [4] Sally Dawson, Howard E. Haber:**A Primer on Higgs Boson Low-Energy Theorems**, by Sally Dawson and Howard E. Haber, CONF-8901127-2
- [5] Marcela Carena, Ian Low, and Carlos E. M. Wagner:**Implications of a Modified Higgs to Diphoton Decay Width**, arXiv:1206.1082
- [6] Bogdan A. Dobrescu, Graham D. Kribs and Adam Martin:**Higgs Underproduction at the LHC**, arXiv:1112.2208v2
- [7] Brian Batell, Stefania Gori, Lian-Tao Wang: **Exploring the Higgs Portal with 10fb^{-1} at the LHC**, arXiv:1112.5180v2
- [8] **Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV**, ATLAS-CONF-2015-044.
- [9] Andy Buckley, James Ferrando, Stephen Lloyd, Karl Nordstrom, Ben Page, Martin Rufenacht, Marek Schonherr, Graeme Watt:**LHAPDF6: parton density access in the LHC precision era**, arXiv:1412.7420v2

Appendix A

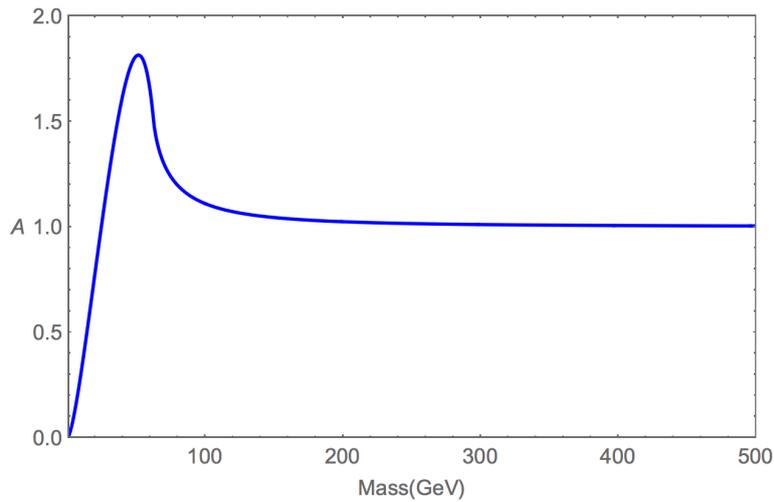
Function A

The function A is defined by low energy theorem for Higgs boson. We choose the light fermion case because Higgs boson mass is discovered at 125 GeV by CERN. Top quark mass is equal to 175 GeV. The function parameter is $4m_i^2/m_H$. m_H is Higgs boson mass.

$$f(\tau) = \begin{cases} \arcsin^{-1}(\sqrt{\tau}) & \text{for } \tau \leq 1 \\ \frac{-1}{4} [\log(\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}}) - i\pi] & \text{for } \tau > 1 \end{cases}$$

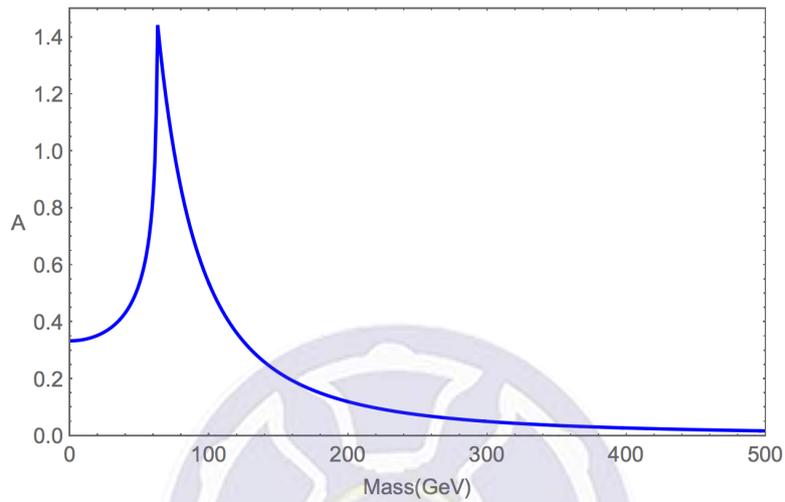
Function $A_{1/2}$

$$A_{\frac{1}{2}}(\tau) = 2\tau[1 + (1 - \tau)f(\tau^{-1})]$$



Function A_0

$$A_0(\tau) = -[\tau - \tau^2 f(\tau^{-1})]$$



Function A_1

$$A_1(\tau) = -[1 + 3\tau + 3(2\tau - \tau^2)f(\tau^{-1})]$$

