1 Introduction

Let K be a field and $R = K[X]$ be the polynomial algebra generated by the entries of a generic $m \times n$ matrix $X = (x_{ij})$ over K. Let p be a positive integer. Let G_p be the set of all p-minors of X and I be the ideal generated by G_p. It is shown by Sturmfels [6] and independently by Caniglia et al. [2] that G_p is a Gröbner basis for I with respect to some lexicographical term order of R (see Theorem 3.1). Later in 1992, Herzog and Trung [5] improved this fact as follows.

Theorem 1.1 *Let $X = (x_{ij})$ be a generic $m \times n$ matrix over a field K, and let $R = K[X]$. Let $a_1 < \cdots < a_r \leq m$, $b_1 < \cdots < b_r \leq n$ be positive integers. Let $D_t(X)$ be the part of the matrix X consisting of the first $a_t - 1$ rows and the first $b_t - 1$ columns. Let $G_t(X)$ be the set of all t-minors of $D_t(X)$, $t = 1, \cdots, r$ and set $G_{r+1}(X)$ be the set of all $(r+1)$-minors of X. Let I be the ideal of R generated by the $G(X) = \cup_{t=1}^{r+1} G_t(X)$; then $G(X)$ is a Gröbner basis for I with respect to the lexicographic term order induced from the variable order*

$$x_{11} > x_{12} > \cdots > x_{1n} > x_{21} > \cdots > x_{m1} > \cdots > x_{mn}.$$

For a symmetric matrix, a similar result is obtained by Conca [3].

Theorem 1.2 *Let $Y = (y_{ij})$, with $y_{ij} = y_{ji}$, be an $n \times n$ symmetric matrix of indeterminates, and let $R = K[Y]$ be a polynomial ring over a field K. Let $a_1 < \cdots < a_r \leq n$ be positive integers. Let $D_t(Y)$ be the part of the matrix Y*

consisting of the first $a_t - 1$ rows and columns. Let $G_t(Y)$ be the set of all t-minors of $D_t(Y)$, $t = 1, \cdots, r$, and set $G_{r+1}(Y)$ be the set of all $(r+1)$-minors of Y. Let I be the ideal of R generated by the $G(Y) = \cup_{t=1}^{r+1} G_t(Y)$; then $G(Y)$ is a Gröbner basis for I with respect to the lexicographic term order induced from the variable order

$$y_{11} > y_{12} > \cdots > y_{1n} > y_{22} > \cdots > y_{2n} > \cdots > y_{nn}.$$

In this paper, we get some results similar to Theorem 1.1 and Theorem 1.2 as follows.

Theorem 1.3 *Let $X = (x_{ij})$ be a generic $m \times n$ matrix over a field K, and let $R = K[X]$. Let $m \geq a_1 \geq \cdots \geq a_r$, $b_1 \leq \cdots \leq b_r \leq n$ be nonnegative integers, and $\eta_1, \cdots, \eta_{r+1}$ be positive integers. Let $D_t(X)$ be the part of the matrix X consisting of the last a_t rows and the first b_t columns. Let $G_t(X)$ be the set of all η_t-minors of $D_t(X)$, $t = 1, \cdots, r$ and set $G_{r+1}(X)$ be the set of all η_{r+1}-minors of X. Let I be the ideal of R generated by the $G(X) = \cup_{t=1}^{r+1} G_t(X)$; then $G(X)$ is a Gröbner basis for I with respect to the lexicographic term order induced from the variable order*

$$x_{11} > x_{12} > \cdots > x_{1n} > x_{21} > \cdots > x_{m1} > \cdots > x_{mn}.$$

Theorem 1.4 *Let $Y = (y_{ij})$, with $y_{ij} = y_{ji}$, be an $n \times n$ symmetric matrix of indeterminates, and let $R = K[Y]$ be a polynomial ring over a field K. Let $n \geq a_1 \geq \cdots \geq a_r$, $b_1 \leq \cdots \leq b_r \leq n$ be nonnegative integers and $\eta_1, \cdots, \eta_{r+1}$*

be positive integers. Let $D_t(Y)$ be the part of the matrix Y consisting of the last a_t rows and first b_t columns. Let $G_t(Y)$ be the set of all η_t-minors of $D_t(Y)$, $t = 1, \cdots, r$, and set $G_{r+1}(Y)$ be the set of all η_{r+1}-minors of Y. Let I be the ideal of R generated by the $G(Y) = \cup_{t=1}^{r+1} G_t(Y)$; then $G(Y)$ is a Gröbner basis for I with respect to the lexicographic term order induced from the variable order

$$y_{11} > y_{12} > \cdots > y_{1n} > y_{22} > \cdots > y_{2n} > \cdots > y_{nn}.$$