Chapter 2

Study of Single Mode Rib Waveguide Based on SOI Substrate and Erbium Doped Waveguide Amplifier

As mentioned in chapter 1, the integrated optical waveguide devices has becoming the trend in the future. In this Chapter we propose a single channel erbium-doped waveguide amplifier (EDWA) building on silicon-on-insulator (SOI) wafers with different length of waveguide and pump power by opti-system. This chapter is arranged as follows: Section 2-1 introduces single mode rib waveguide and Section 2-2 introduces the technique of optical Amplifier. The advantages of EDWA applied in integrated optics are presented and discussed. We also present the principal and mathematical formulations of EDWA in Section 2-3. Section 2-4 shows the Simulation and Analysis of Erbium Doped Waveguide Amplifier. Finally, we provide a summary in section 2-5.

2-1 Single Mode Rib Waveguide

Single mode propagation is an important requirement in optical devices. It was first reported by Peterman [34] in 1976, he uses Mode-matching and BPM to analyze semiconductor rib waveguide and then apply to silicon rib waveguide. Equation (2-1) and (2-2) give the condition to design a single-mode waveguide for SOI structure.

$$\frac{W}{H} \le \alpha + \frac{\frac{h}{H}}{\sqrt{1-\left(\frac{h}{H}\right)^2}} \qquad (2-1)$$

$$\frac{h}{H} \ge 0.5 \qquad (2-2)$$

As shown in Fig. 2-1, the single mode rib waveguide of 2×2 MMI with the width of rib waveguide W = 4μm. h = 2μm is the height of slab waveguide and H = 2.5μm is the total rib height. α = 0.3 is a dimensionless constant. The refractive index of the rib waveguide is 3.45 and the cladding layer is controlled at 1.48 through the photolithography process. The rib waveguide is designed by the single mode formula as shown in equation (2-1) and equation (2-2). The mode pattern of the single rib waveguide is shown in Fig. 2-2.

2-2 Basics of Optical Amplifiers

Laser material is fluorescent, and fluorescence is the key to understand optical amplifiers. As a matter of fact, when searching for a new laser material, the fluorescent properties of the material are the first thing to be checked. The illuminating light is called the pump light. Even though fluorescence begins as soon as the pump light is turned on, when the pump light is turned off, the fluorescence is sustained. It decays with a characteristic lifetime. From these observations, scientists have found an explanation using the concept of energy levels and transitions between energy levels. The model shown in Fig. 2-3 will be used as a starting point. This model is known as the three-energy-level model. Pumping takes place between energy levels E_1 and E_3, where $h\nu_{31}=E_3 - E_1$ and ν_{31} is the frequency of the pump light. The fluorescent glow takes place between E_2 and E_1, where $h\nu_{21}=E_2 - E_1$ and ν_{21} is the frequency of the glow light [52].

Energy level E_3 is actually a bundle of many closely spaced energy levels, rather than one discrete level. When the pump light is turned on, upward transitions from E_1 to the E_3 band take place, provided the energies contained in the pump light match the E_1 to E_3 band transition. Almost immediately, downward transitions are initiated between the closely spaced E_3 levels, as well as from E_3 to the nearby level E_2. Because of the narrow spacing, the transitions between the E_3 levels are primarily associated with phonons and nonradiative transitions and occur very quickly over lifetimes of the order of femtoseconds to nanoseconds. The released energy is converted into crystal lattice vibrations or phonons with energy $hv_{phonon} = E_{n+1} - E_n$.

According to Fig. 2-3, there is a large energy gap between E_2 and E_1, which means photons are involved, and not phonons. The downward transition from E_2 to E_1 is responsible for the fluorescent glow. We are particularly interested in materials where E_2 is a metastable state, which means the downward transition from E_2 to E_1 occurs over a lifetime τ_2 of milliseconds to hours. Thus, $\tau_2 > \tau_3$ and the glow lasts a long time after the pump has been turned off.

E_1 and E_2 are discrete levels, whereas E_3 is a band of levels. Hence, the wavelength of the fluorescent glow is very specific (E_2 to E_1 transition), but bands of wavelengths will work for the pump light (E_1 to E_3 band transition).

When the transition from E_2 to E_1 occurs spontaneously, a photon of energy hv_{21} is released. This process is called spontaneous emission. Spontaneously emitted photons not only travel in different directions but also have different phases. These photons are said to be incoherent.

There is another important mechanism for the emission of light that is called stimulated emission. Stimulated emission will be explained by referring once again to the three-level model in Fig.2-3. The pump light causes the population of atoms in level E_3 to increase. This population increase in E_3, however, is quickly transferred to that of E_2 because of the fast decay from E_3 to E_2, finally resulting in the population buildup in E_2 because of long τ_2. If the material is illuminated by light of frequency while this buildup of population in E_2 exists, a significant increase in the light intensity is observed at frequency v_{21}. The amount of the increase is proportional to the v_{21} illumination.

Let's take a closer look at what is going on. Before the buildup in level E_2 has a chance to decay spontaneously, the input photons of energy hv_{21} come along. Because these photons happen to have the right frequency, they induce downward transitions from E_2 to E_1. For each induced downward transition, a photon of energy hv_{21} is released. Furthermore, both the released photon and the photon that induced the release are not only identical in frequency but also in phase and direction. Hence, the v_{21} stimulated emission is coherent with the v_{21} illuminating light [35].

Stimulated emission is the basis for lasers and optical amplifiers. In a laser, photons capable of causing stimulated emission make multiple passes through a laser cavity, inducing the release of photons of identical energy with each pass. A coherent beam of collimated light with a narrow frequency bandwidth is created. In an optical amplifier, the photons capable of causing stimulated emission (supplied by the signal light) make a single pass through the material. The power of the signal light is amplified by the stimulated emission while traveling through the amplifier.

Emission and the reverse process of absorption can be interpreted as a kind of resonance phenomenon taking place between level E_1 and E_2. A photon is emitted whenever there is a downward transition from E_2 to E_1, and a photon is absorbed whenever there is an upward transition from E_1 to E_2. Moreover, their transition probabilities are identical.

If there is no signal light at frequency v_{21}, then the emission is mainly spontaneous emission. When both a population inversion and input signal light of the right frequency v_{21} are presented, then the output light due to the stimulated emission grows in proportion to the strength of the input signal light, with the population inversion as a proportional constant.

In the amplifier, there exists not only stimulated emission that is used for amplification, but also spontaneous emission that harms the quality of the amplifier. Spontaneous emission is independent of the input signal light and causes noise problems. Indeed, this noise source is a major concern when using an optical amplifier. Choosing an amplifying medium with a long lifetime is important because the probability of spontaneous emission is $1/\tau_{21}$.

An example of an amplifier that corresponds to the three-level model is the erbium-doped fiber amplifier. This amplifier is used at λ=1.55 μm. Not all amplifiers are based on the three-level model. An example of an amplifier based on a four-level model is the neodymium (Nd)-doped fiber amplifier, which is useful at λ=1.06 and 1.32 μm.

The four-level model has an extra level E_0 below E_1. E_0 is the ground level. The pumping takes place between E_0 and E_3. The population inversion is between E_2 and E_1. As before, the lifetime τ_{32} is short and the lifetime τ_{21} is long. The pumping action populates E_2 via the fast decay from E_3. Compared to the three-level model, the four-level model has a larger population inversion between E_2 and E_1 because E_1 is very quickly emptied out into E_0, which is constantly being evacuated by the pump light. The population inversion between E_2 and E_1 is not only larger but is also less sensitive to the condition of the pump light.

With the three-level material, however, the population in E_1 depends on the strength of the pumping action and its ability to evacuate level E_1. Consequently, the population inversion between E_2 and E_1 is more dependent on the pump power and likewise, the gain of the amplifier is more dependent on the pump power [52].

2-3 Introduction of the Technique of Erbium Doped Waveguide Amplifier

Silicon is an ideal material for the fabrication of optical waveguides that are compatible with optical communication technology at 1.5 μm, because of its high transparency and high refractive index at this wavelength [36]. Erbium is a rare earth element belonging to the group of Lanthanides. When embedded in a solid, erbium generally assumes the trivalent Er^{3+} state, which has an electronic configuration [Xe]-4f ^{11}. The Er^{3+} ion has an incompletely filled 4f-shell, allowing for different electronic configurations with different energies due to spin-spin and spin-orbit interaction [37]. Optical amplification with Erbium-doped glass as the gain medium has been a key enable for Dense Wavelength Division Multiplexed (DWDM) optical transport systems. Leveraging on the fundamental properties of Erbium in a glass host, the Erbium Doped Fiber Amplifier (EDFA) has demonstrated high gain, low noise, and full compatibility with DWDM signals. Erbium Doped Waveguide Amplifiers (EDWAs) offer these functionalities at a low price per function, and over the past few years, this has driven EDWA-based devices from research labs to commercialization [38]. Erbium doped materials are of great interest in optical communications technology, as they can serve as the gain medium in lasers and optical amplifiers operating at the standard telecommunications wavelength of 1.5 μm [39,40]. Erbium Doped Optical Planar Waveguide Amplifier have broad application on 1.5μm optical communication. These devices have the advantage of high gain, low pump power, small device size. It can serve, for example, to compensate for coupling losses, waveguide losses, or the power division in optical splitters [41]. Our

Erbium doped waveguide used two kinds of waveguide about Arrayed Waveguide Grating (AWG) and multimode Interference (MMI) to measure the relationship between pump power and gain in the device. MMI splitter aleviates the difficulties associated with concatenation of the Y junction. The MMI splitter consists of a single- mode input port and several single-mode output ports with a multimode region in the middle section. The parameters can be designed for low loss as well as for a balanced division of power in the output ports [42, 43].We try to implant ions of erbium into our 2x2 MMI to research the variation of the device and expect the combination of EDWA and WDM elements in Chapter 4.

2-3-1 Gain of Optical Planar Waveguide Amplifier

The gain of an optical amplifier only depends on the transitions between the two levels included in the population inversion. In the discussion of fluorescence, the levels involved in the population inversion were distinctly well-defined levels. However, the description can easily be extended to a population inversion between two bands of energy levels. The transitions involve ions, molecules, electrons or atoms in the energy bands depend on the material. The term “carrier” will be used to represent an atom, molecule, ion, or electron in any particular case. We consider the two energy levels E_2 and E_1 shown in Fig.2-4, ignoring pump level E_3. Energy is released in a downward transition of a carrier, and energy is absorbed in an upward transition. Stimulated transitions are the transitions originated by the existence of an external photon and take place in both directions. In stimulated absorption, the external photon is absorbed as the upward transition takes place, and in stimulated emission, the external photon causes the release of a photon of identical energy as the downward transition takes place. Spontaneous emission is the release of a photon in a downward transition that happens voluntarily, without any external photon. There is nothing about spontaneous absorption, or a spontaneous upward transition [44].

We will calculate the rate of transitions of the carriers. If we let N_2 represent the number of carriers per unit volume in E_2, and N_1 represent the same in energy level E_1, we can find the number of carrier transitions per second per unit volume due to spontaneous emission is [45]

$$-\frac{dN_2}{dt}_{spon} = AN_2 \qquad (2\text{-}3)$$

A is Einstein's A coefficient.

$$A = \frac{1}{\tau_{21}} = \frac{1}{\tau_{spon}} \qquad (2\text{-}4)$$

A is equal to the inverse of τ_{21}. The rate of change in N_2 per unit time only due to stimulated processes is the difference between the number of carriers that undergo absorption by making the upward transition from E_1 to E_2 and the number of carriers that experience stimulated emission by taking the downward transition from E_2 to E_1. The induced transition probabilities are identical for both upward and downward transitions and are represented by one probability W_s. We can obtain the net number of carriers making the downward transition per second per unit volume due to stimulated emission and shown in eq. (2.5)

$$-\frac{dN_2}{dt}_{stim} = W_s(N_2 - N_1) \qquad (2\text{-}5)$$

Because of the value of W_s is proportional to the light energy density E_d of the stimulating light, so we can obtain this equation:

$$W_s = BE_d \qquad (2\text{-}6)$$

where the B constant is known as Einstein's B coefficient. E_d is related to the light intensity:

$$E_d = (\frac{I}{\upsilon}) \qquad (2\text{-}7)$$

I is the light intensity and υ is the velocity in one second. By including the spectral lineshape function $g_t(v)$, W_s can be rewritten as [46,47]

$$W_s(v) = B\frac{g_t(v)}{\upsilon}I \qquad (2\text{-}8)$$

To continue, Einstein's A and B coefficients are related to each other. Einstein derived the relationship between A and B from the equilibrium condition of blackbody radiation [46] as

$$\frac{A}{B} = \frac{8\pi n_1^3}{c^3} h v^3 \qquad (2\text{-}9)$$

n_1 is the index of refraction of a blackbody radiator and c is the speed of light. When we Insert Equation (2-4) and (2-9) into Equation (2-8), W_s will becomes

$$W_s(v) = \sigma_s \frac{I}{hv} \qquad (2\text{-}10)$$

and

$$\sigma_s = \frac{\lambda^2 g_t(v)}{8\pi n_1^2 \tau_{spon}} \qquad (2\text{-}11)$$

And the quantity σ_s is stimulated emission cross section.

Let us rewrite Equation (2-9) noting the fact that there are $(\frac{8\pi n_1^3 v^2}{c^3})\Delta V \cdot \Delta v$ modes in the frequency range between $v + \Delta v$ and v in the volume of a blackbody radiator. With this fact, Eq. (2-9) is described as

$$\frac{A}{B} = m(v) h v \qquad (2\text{-}12)$$

Where

$$m(v) = \frac{8\pi n_1^3}{c^3} v^2 \tag{2-13}$$

Equation (2-13) means the number of modes per unit volume
per unit frequency. Then we insert Equation (2-13) into Equation (2-11) or inserting Equation (2-12) into Equation (2-8) and Equation (2-10) we can get:

$$\sigma_s = \frac{A g_t(v)}{m(v)\upsilon} \tag{2-14}$$

Equation (2-10) can be interpreted as follows the photon flux density F is the number of photons passing through unit area per second. Because $\frac{I}{hv}$ is the photon flux density F, so we can let Equation (2-5) become

$$\begin{aligned} \frac{dN_2}{dt} &= W_s(N_2 - N_1) \\ &= \sigma_s \frac{I}{hv}(N_2 - N_1) \\ &= \sigma_s F(N_2 - N_1) \end{aligned} \tag{2-15}$$

The value of σ_s depends on the host material and the wavelength and the peaks are in the proximity of 1540 nm, and σ_s tapers off in 1540 ± 40 nm [48].

Then let us consider the specific case of an EDWA. Imagine a fictitious open sided box, such as shown in Fig. 2-5, moving with the photons through a number of Er^{3+} ions without friction. The increase in the light energy per unit time inside the box is calculated. If each photon having energy is hv, the total contributions from both stimulated and spontaneous emissions are combined from Equation (2-3) and (2-5), we can obtain the Equation (2-16) as follows:

$$\frac{dE_d}{dt} = \left[(N_2 - N_1)\sigma_s \frac{I}{hv} + AN_2 \right] hv \qquad (2\text{-}16)$$

From Equation (2-7)

$$\frac{dE_d}{dt} = \frac{dI}{\upsilon dt} = \frac{dI}{dz} \qquad (2\text{-}17)$$

and Eq. (2-16) becomes

$$\frac{dI}{dz} = (N_2 - N_1)\sigma_s I + AN_2 hv \qquad (2\text{-}18)$$

We can let

$$\begin{aligned} (N_2 - N_1)\sigma_s &= g \\ AN_2 hv &= H \end{aligned} \qquad (2\text{-}19)$$

The solution of this differential equation will be found and Equation (2-16) becomes

$$\frac{dI}{dz} = gI + H \qquad (2\text{-}20)$$

The solution becomes

$$I = I_s e^{gz} + \frac{H}{g}(e^{gz} - 1) \qquad (2\text{-}21)$$

Inserting Eq. (2-19) into (2-21) gives the output light intensity I at z = L
We can find:

$$I = GI_s + (G-1)\frac{N_2}{N_2 - N_1}\frac{Ahv}{\sigma_s} \qquad (2\text{-}22)$$

Where

$$G = e^{gL} \qquad (2\text{-}23\ )$$

To determine the value of σ_s for an erbium doped waveguide the first step toward is to find the value of $m(v)$ specifically for a waveguide.
The intensity of the transmitted beam can be written as [49]

$$I(O) = cI_0 \times e^{-(\alpha + \sigma_{abs} N \Gamma)L} \qquad \text{pump off } (2\text{-}24)$$

$$I(P) = cI_0 \times e^{-[\alpha + (\alpha_{abs} N_1 - \sigma_e N_2)\Gamma]L} \qquad \text{pump on } (2\text{-}25)$$

where c is the coupling efficiency, I_0 is the input signal intensity, α is the waveguide loss, α_{abs} is the absorption cross section, σ_e is the emission cross section, N is total doping concentration, Γ is the core-mode overlap, and L is the illuminated length. N_1 and N_2 are the concentration of Er^{3+} ions in the grounded and excited states, respectively, such that $N_1+N_2=N$. The coupling efficiency is difficult to estimate, as can be seen in Fig 2-6. Furthermore, due to the experimental nature of the fabrication process, α is also expected to be high. Therefore, we concentrate on signal enhancement (SE), defined as :

$$SE \equiv I(P)/I(O) = e^{2(\sigma N_2 \Gamma)L} \tag{2-26}$$

In this equation, we have approximated that $\sigma_{abs} = \sigma_e = \sigma$, which is quite accurate for 1535nm [50]. If there is very little upconversion in our Erbium doped waveguide, Er^{3+} can be modeled as a simple two level system [51].

$$\frac{dN_2}{dt} = \Sigma\Phi(N - N_2) - wN_2 \tag{2-27}$$

Σ is the effective absorption cross section of Er, Φ is the pump flux, and w is the decay rate of excited Er^{3+}. In a low signal power, we can neglect the stimulated emission. Thus, we can get the equation:

$$SE(P) = e^{2[\sigma N \Sigma\Phi/(\Sigma\Phi + w)\Gamma]L} \tag{2-28}$$

Let us consider L as the entire waveguide length, s is a cross section, and there are reflectors at each end. The longitudinal modes that are generated by two oppositely traveling waves in the cavity have to satisfy the condition of [52]

$$\frac{\lambda_p}{2} p = L \tag{2-29}$$

In the waveguide amplifier, there is no reflection and no longitudinal modes, but in order to match the classical approach, we assume there are modes due to the reflected and forward waves. In the end, only one-half of the modes will be used because of the absence of the reflected wave in the waveguide amplifier, where λ_p is the wavelength of the pth longitudinal mode. In terms of the frequency f_p, Equation (2-29) is written as

$$f_p = \frac{\upsilon}{2L} p \tag{2-30}$$

where υ is the phase velocity in the core. That of the (p +1)st mode is

$$f_{p+1} = \frac{\upsilon}{2L}(p+1) \tag{2-31}$$

So we can get the separation between adjacent modes:

$$f_{p+1} - f_p = \frac{\upsilon}{2L} \tag{2-32}$$

The number of modes per unit frequency in this cavity is

$$\frac{1}{f_{p+1} - f_p} = \frac{2L}{\upsilon} \tag{2-33}$$

Only the modes that carry the energy forward will be considered. We also have to account for the number m_t of transverse modes. For a circular polarized wave, which has two orthogonal modes, m_t is 2. And for a linearly polarized mode, m_t is 1. The number $m(v)$ per unit volume is obtained:

$$m(v) = m_t \frac{L}{\upsilon} \frac{1}{Ls} = m_t \frac{1}{\upsilon s} \tag{2-34}$$

Now, the transition cross section σ_s is found by combining Equation (2-14) and (2-34):

$$\sigma_s = \frac{Asg_t(v)}{m_t} \tag{2-35}$$

Inserting Equation (2-35) into Equation (2-22) and integrating across the cross section of the waveguide gives

$$P = GP_s + (G-1)n_{spon}m_t hv\Delta v_t \qquad (2\text{-}36)$$

P and P_s are the output and input light powers.

$$n_{spon} = \frac{N_2}{N_2 - N_1} \qquad (2\text{-}37)$$

The first term of Equation (2-36) is the amplified signal power and the second term is the amplified spontaneous emission (ASE) noise. The factor n_{spon} is called the population inversion factor. If we increase the gain factor g of the amplifier, we must increase the population difference N_2- N_1. The ASE noise can be lowered by reducing n_{spon}, but n_{spon} cannot be made zero by making N_2=0 because the amplification also disappears. A more appropriate minimum value of n_{spon} is unity when N_1=0 and N_2=N with N =N_1+N_2. The value of n_{spon} cannot be smaller than unity even for the lossless case. As seen from Equation (2-36), this noise is amplified by (G-1) with almost the same gain as that for the signal. It is essential for an optical amplifier to be provided with an optical filter to repress the ASE noise. The equivalent circuit derived from Equation (2-36) is shown in Fig. 2-7.

2-3-2 RATE EQUATION FOR THE THREE-LEVEL MODEL OF Er^{3+}

In the preceding paragraph, we looked at the expression for the amplifier gain, which was derived from the rate equations for the two levels E_2 and E_1 involved in the population inversion. This section we search the solutions of the rate equation for all three levels of a three-level model [53]. The purpose of this analysis is to exhibit the relation of the threshold and saturation of the gain with respect to the pump light. The example which we choose is the three-level Er^{3+} doped waveguide amplifier, which is shown in Fig. 2-8. This figure indicates the following quantities: the number of carriers N_i per unit volume in the energy level E_i ; the spontaneous transition lifetime τ_{ij} which is the inverse of the rate of the transition from the I^{th} energy level to the j^{th} energy level (where i > j); the stimulated transition probability W_p between E_3 and E_1; and the stimulated transition probability W_s between E_2 and E_1. The quantity W_p is defined similar to Equation (2-10) as follows [54]:

$$W_p(v_p) = \sigma_p \frac{I_p}{hv_p} \tag{2-38}$$

The rates of difference in the populations of the three levels are

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - \frac{N_3}{\tau_3} \tag{2-39}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - W_s(N_2 - N_1) \tag{2-40}$$

$$\frac{dN_1}{dt} = -W_p(N_1 - N_3) + \frac{N_2}{\tau_{21}} + W_s(N_2 - N_1) \tag{2-41}$$

and

$$\frac{1}{\tau_3} = \frac{1}{\tau_{32}} + \frac{1}{\tau_{31}} \qquad (2\text{-}42)$$

The simultaneous rate equations will be solved for the steady-state and we will find the population ratio, the population difference, the gain, and the saturation gain of the amplifier.

2-3-2-1 Normalized Steady-State Population Difference

The population difference between the E_1 and E_2 levels, which determines the gain of the amplifier, will be found. Since [55]

$$\tau_{31} > \tau_{32} \qquad (2\text{-}43)$$

and with Equation (2-42) to (2-43), Equation (2-39) becomes

$$W_p(N_1 - N_3) = \frac{N_3}{\tau_{32}} \qquad (2\text{-}44)$$

Putting Eq. (2-44) into (2-40) gives

$$0 = W_p(N_1 - N_3) - \frac{N_2}{\tau_{21}} - W_s(N_2 - N_1) \qquad (2\text{-}45)$$

E_2 and E_3 levels of the Er^{3+} doped waveguide amplifier pumped by 1.48-μm pump light are very closely spaced, and because of the fast relaxation process, the population ratio between these two levels quickly reaches the Boltzmann population ratio [47]:

$$\beta = \frac{N_3}{N_2} = e^{-(\Delta E / kT)} \tag{2-46}$$

At room temperature, $\beta = 0.38$. Putting Equation (2-46) into (2-45) gives

$$\frac{N_2}{N_1} = \frac{W_p + W_s}{W_p \beta + 1/\tau_{21} + W_s} \tag{2-47}$$

Let the numerator and denominator of Equation (2-47) be A and B,* respectively. N is defined as $N=N_1+N_2$ and N becomes

$$N = N_1 + \frac{A}{B} N_1 \tag{2-48}$$

and

$$N_1 = \frac{B}{A+B} N \tag{2-49}$$

$$N_2 = \frac{A}{A+B} N \tag{2-50}$$

Equations (2-49) and (2-50) can be rewritten as

$$\frac{N_2 - N_1}{N} = \frac{A - B}{A + B} \tag{2-51}$$

Putting the numerator and denominator of Equation (2-47) into this equation and letting $\tau_{21} = \tau$ gives

$$\frac{N_2 - N_1}{N} = \frac{(1-\beta)W_p\tau - 1}{(1+\beta)W_p\tau + 2W_s\tau + 1} \tag{2-52}$$

$W_p\tau$ is called the normalized pumping rate.

Equation (2-10) and (2-38) we can find $W_s\tau$ and $W_p\tau$ are quantities representing the signal and pump light intensities, respectively. (N_2-N_1) represents the gain factor from Equation (2-19). Thus, Equation (2-52) is an important equation that relates signal and pump powers to the gain of the amplifier. In Fig. 2-9, N_2-N_1/N is plotted as a function of the pump power $W_p\tau$ with the signal power $W_s\tau$ and β as parameters.

2-3-2-2 Gain of the Amplifier

From Equations (2-19) and (2-52), we can realize the gain factor of the optical amplifier and the criteria for whether or not the amplifier experiences gain are [57]

$$\frac{N_2 - N_1}{N} \begin{cases} > 0 & gain \\ = 0 & transparent \\ < 0 & loss \end{cases} \tag{2-53}$$

When gain occurs in one medium, the medium is said to become active. Equation (2-52) means that the medium does not become active until it reaches the threshold value of [72]

$$W_p^{th}\tau = \frac{1}{1-\beta} \tag{2-54}$$

This threshold occurs at the same value regardless of the signal power level. Equation (2-54) with $\beta = 0$ reduces to

$$W_p^{th} = \frac{1}{\tau} \tag{2-55}$$

When the pump power is just enough to support the carriers for spontaneous emission, we call that condition the threshold condition. From Equation (2-54), we can find out if the lifetime τ of the metastable state of erbium increases whether the threshold pump power will decrease. This is an additional worth about long lifetime besides reducing spontaneous emission noise. As seen from Equation (2-52), the maximum of population difference will become when

$$2W_s\tau = 0 \tag{2-56}$$

Let us define the maximum population difference ΔN_{max} from integrating Equations (2-52) and (2-56):

$$\frac{\Delta N_{max}}{N} = \frac{(1-\beta)W_p\tau - 1}{(1+\beta)W_p\tau + 1} \qquad (2\text{-}57)$$

Inserting Equation (2-57) into (2-52) we can give

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{max}}{N} \frac{(1+\beta)W_p\tau + 1}{(1+\beta)W_p\tau + 2W_s\tau + 1} \qquad (2\text{-}58)$$

Or

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{max}}{N} \frac{1}{1 + \dfrac{2W_s\tau}{(1+\beta)W_p\tau + 1}} \qquad (2\text{-}59)$$

When the value of $W_s\tau$ (signal power) reduces N_2 - N_1 to one-half of ΔN_{max}, we can indicated it as W_s^{sat}.

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{max}}{N} \frac{1}{1 + \dfrac{W_s}{W_s^{sat}}} \qquad (2\text{-}60)$$

where

$$W_s^{sat} = \frac{1}{2}\left[(1+\beta)W_p + \frac{1}{\tau}\right] \tag{2-61}$$

Inserting Equation (2-54) into Equation (2-61), it will becomes

$$W_s^{sat} = \frac{1}{2\tau}(1 + \frac{1+\beta W_p}{1-\beta W_p^{th}}) \tag{2-62}$$

Inserting Equation (2-10) into Equation (2-60), we can find:

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{\max}}{N} \frac{1}{1 + I_s / I_s^{sat}} \tag{2-63}$$

The above analysis shows several important conclusions. First, the gain factor of the amplifier is proportional to the population difference. Second, an increase in the pump power results in an increase not only in the signal saturation intensity (Equation (2-62)) but also in the gain of the amplifier (Equation (2-59)). Finally, the signal power increases, the gain of the amplifier gradually decreases and finally reaches zero (Equation (2-63)).

2-3-3 Advantages and Disadvantage between 1.48μm and 0.98μm pump light

Fig. 2-10 shows the energy level diagram of Er^{3+} [58]. The types of lasers that may be used as the pump light are indicated in the chart. When the absorption coefficient is higher, the absorption cross section σ_p is larger, and it is easier to pump. Solid state devices are practical for optical communication because they have the advantage of reliability, longevity, and compact size, which means the surplus pump source are 800 nm, 980 nm, and 1.48 μm. But the 800nm pump, however, is inefficient due to excited state absorption (ESA). Fig. 2-11 explains the ESA loss. If there is still a higher energy level E_4 and there is a space between E_4 and E_2 is identical with that between E_3 and E_1. Because of E_3-E_1=E_4-E_2, there is a possibility that some of the carriers in level E_2 would be raised further to the E_4 level by the pump light. Now the attractive pump source becomes only 980-nm and 1.48-μm pump light. Comparisons between these two pump wavelengths will be made using the solutions of the rate equations obtained in Section 2-3-2.

First, the values of β for these two pump wavelengths are different. The 1.48-μm pump is special when the ratio N_3/N_2 of the population density is tightly clamped by the Boltzmann distribution because the energy level of the 1.54-μm signal is so close the 1.48-μm pump. At room temperature, β is

$$\beta = \frac{N_3}{N_2} = e^{-\Delta E/KT} = 0.38 \qquad (2\text{-}64)$$

In contrast, β is almost zero for 0.98-μm pump light. This creates differences between 1.48-μm and 0.98-μm pumping. As seen from Equation (2-52), the maximum obtainable population difference for a large normalized pumping rate is

$$\left(\frac{N_2 - N_1}{N}\right)_{\max} = \frac{1-\beta}{1+\beta} \qquad (2\text{-}65)$$

In the case of 1.48-µm pumping, substituting β=0.38 in Equation (2-65) gives

$$\left(\frac{N_2 - N_1}{N}\right)_{\max,1.48-\mu m \quad pump} = 0.45$$

And 0.98-µm pumping, Equation (2-65) becomes

$$\left(\frac{N_2 - N_1}{N}\right)_{\max,0.98-\mu m \quad pump} = 1$$

Thus, the 0.98-µm pump amplifier has better gain than the 1.48-µm pump light. The 0.98-µm pump is also better than the 1.48-µm pump in terms of the spontaneous noise of the amplifier. As got earlier, the amplified spontaneous emission (ASE) noise is proportional to

$$n_{spon} = \frac{N_2}{N_2 - N_1}$$

From Equations (2-49) and (2-50), n_{spon} is rewritten as

$$\frac{N_2}{N_2 - N_1} = \frac{A}{A - B} \qquad (2\text{-}66)$$

and from Eq. (2-47), n_{spon} becomes

$$n_{spon} = \frac{W_p + W_s}{W_p(1-\beta) - 1/\tau} \quad (2\text{-}67)$$

When we increase the pump power to reduce n_{spon} and n_{spon} approaches the limit:

$$n_{spon} = \frac{1}{1-\beta} \quad (2\text{-}68)$$

The 0.98-μm pump can ultimately reduce n_{spon} to 1, but the 1.48-μm pump can only reduce n_{spon} to 1.61.

When we compare the pumping efficiency, which means the gain in dB obtained per milliwatt of pump power, the 0.98-μm pump is also better than the 1.48-μm pump. The threshold pump power is given by Equation (2-54). For 1.48 μm, the threshold is defined as $W_p^{th}\tau$ = 1.61, and for the 0.98-μm pump, the threshold $W_p^{th}\tau$ = 1.0. The gain of the amplifier per unit pump power for a 0.98-μm pump laser is 10 dB /mW, whereas for a 1.48-μm pump laser, it is 5 dB /mW.

We can find the saturation signal power intensity, Equation (2-61) contains a factor (1+β). For the same pump power W_p, the saturation signal power intensity $W_p^{th}\tau$ of the 1.48-μm pump is higher than 0.98-μm pump. Thus, the 1.48-μm pump is better than the 0.98-μm pump in this respect.

In summary, the 0.98-μm pump is better with respect to the gain per unit pump power, the amount of ASE noise, and the threshold pump power, but the 1.48-μm pump is superior with respect to saturation signal power and tolerance of the wavelength of the pump light [59].

2-4 Simulation and Analysis of Erbium Doped Waveguide Amplifier

Though the briefly introduce the operation and principle of EDWA, we also need the result of simulation to analyze our case. This section investigates the simulation method opti-system which allows for the design and simulation of waveguide amplifiers with arbitrary spatial refractive index and doping profiles [60]:

I. Allows for the calculation of the gain and noise characteristics of the high concentration Er^{3+} doped waveguide amplifiers
II. Considers pump excited-state absorption
III. Considers multimode operation for the pump and signals
IV. Homogeneous upconversion (HUC) from $^4I_{13/2}$ to $^4I_{11/2}$ levels
V. Pair-induced quenching—PIQ
VI. Nine energy levels considering double-clad fiber design
VII. Spectral and longitudinal 3D graphs with forward and backward ASE, signals, and pump power.
VIII. Internal mode solver

In this section, we integrated single channel Erbium-doped waveguide amplifier into opti-system software. In our system, we combined the tunable laser light source, EDWA, dual port analyzer and Optical spectrum Analyzer in Fig.2-12. We used the opti-system software to simulate the gain of EDWA in our system. The first case, we set 1532nm light source and changed the length of single channel Erbium-doped waveguide from 2 cm to 10 cm and simulate the gain and noise figure, respectively. The light source average power is small signal with -30dBm and we set the same Er^{3+} dose of $3x10^{14}$ Er/cm^2 and let the pump power upper limit is 400mW with pump power wavelength at 980nm. And we show the result from Fig 2-13. We can find the maximum gain is 7.45 (dB) and with 10cm waveguide and 400mw pump power is shown in Fig. 2-13. The second case we set 1549nm light source and different waveguide length from 10cm to 50cm and find the relation between net gain and waveguide length with several pump power level and shown in Fig 2-14. Finally, we set different waveguide length from 10cm to 50cm and find the relation between noise figure and pump power as shown in Fig. 2-15. The maximum gain with a 28cm waveguide and 400mW pump power is expected to be 10.02 (dB) as shown in Fig. 2-14, and the noise figure is 5.38 (dB) as shown in Fig. 2-15..

2-5 Summary

The EDWA plays an important role in the Dense Wavelength Division Multiplexed (DWDM) optical transport systems [38]. We described the basic operation of the optical amplifier and simulated the single channel erbium-doped rib waveguide with different lengths and several different pump power settings to find the relationship between length and net gain. In this chapter, we introduce the technique of Arrayed Waveguide Grating and optical Amplifier. We also present the principle and mathematical formulations of EDWA and show the simulation of EDWA. From the last simulation of the Opti-system, we can find the maximum net optical gain of 10.02 (dB) is expected for a waveguide length of 28cm with a higher pump power of 400 mW. In conclusion, in accordance with the simulation result, Erbium-doped silicon waveguides operate at the third telecommunication window near 1.5 μm, are attractive due to their small size and potential integration as loss-compensating components with other optical devices, such as passive splitters or combiners.

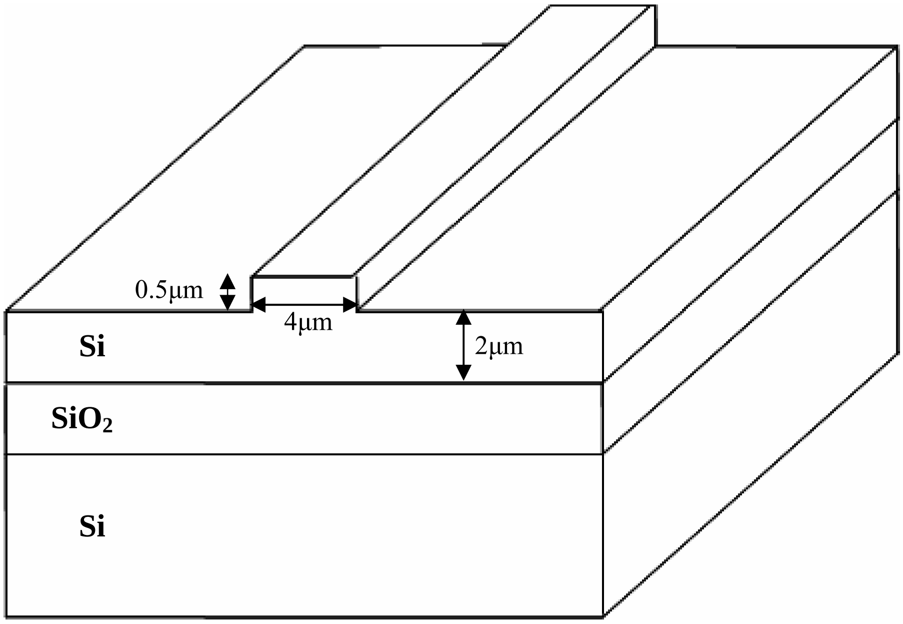

Fig. 2-1 Structure of single mode Silicon on Insulator(SOI) rib waveguide of Multimode Interference(MMI)

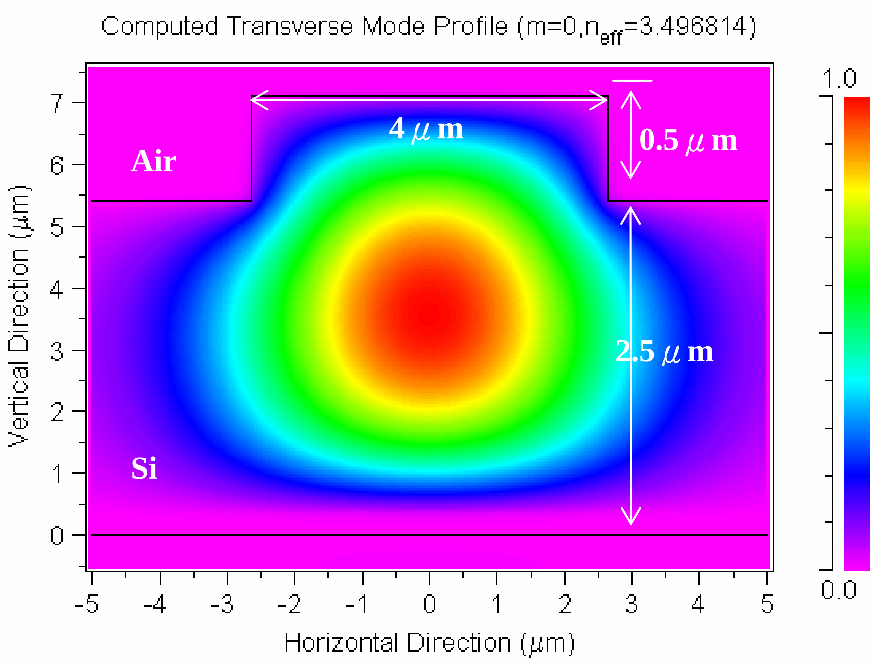

Fig. 2-2 mode pattern of the single rib waveguide

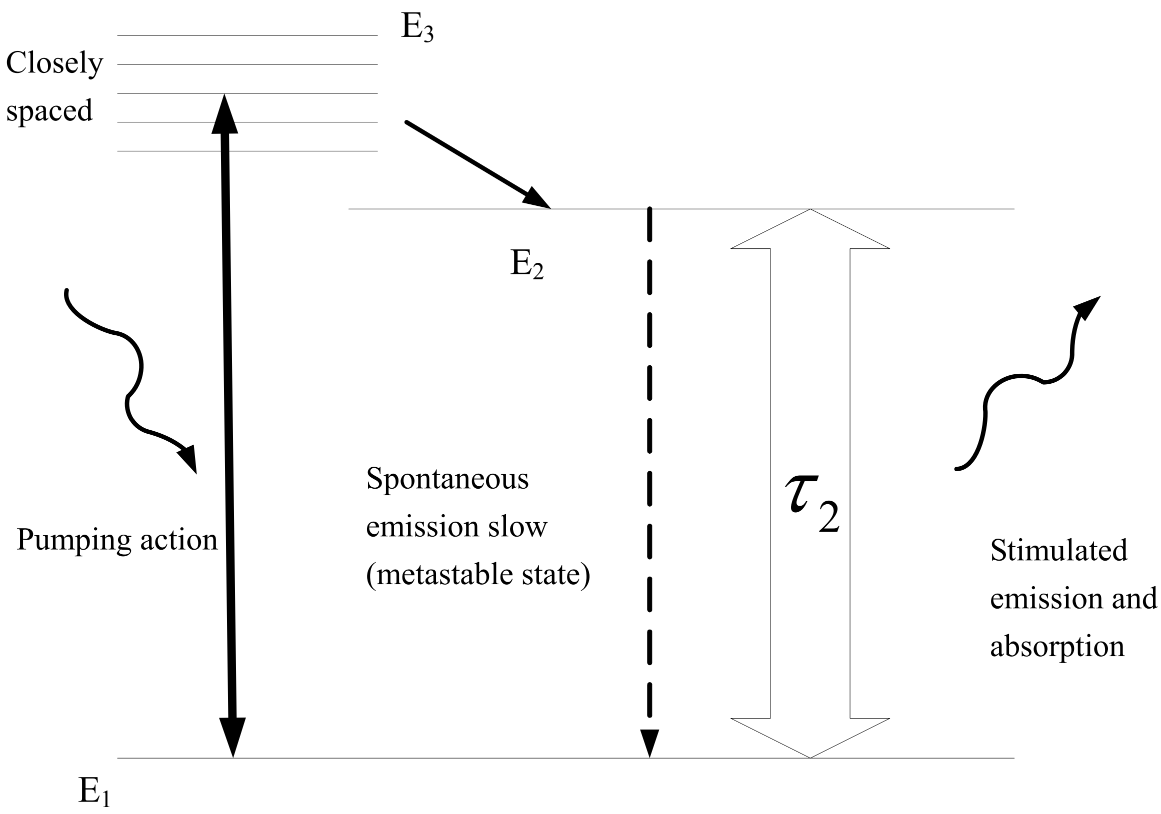

Fig. 2-3 Principle of a three-level amplifier

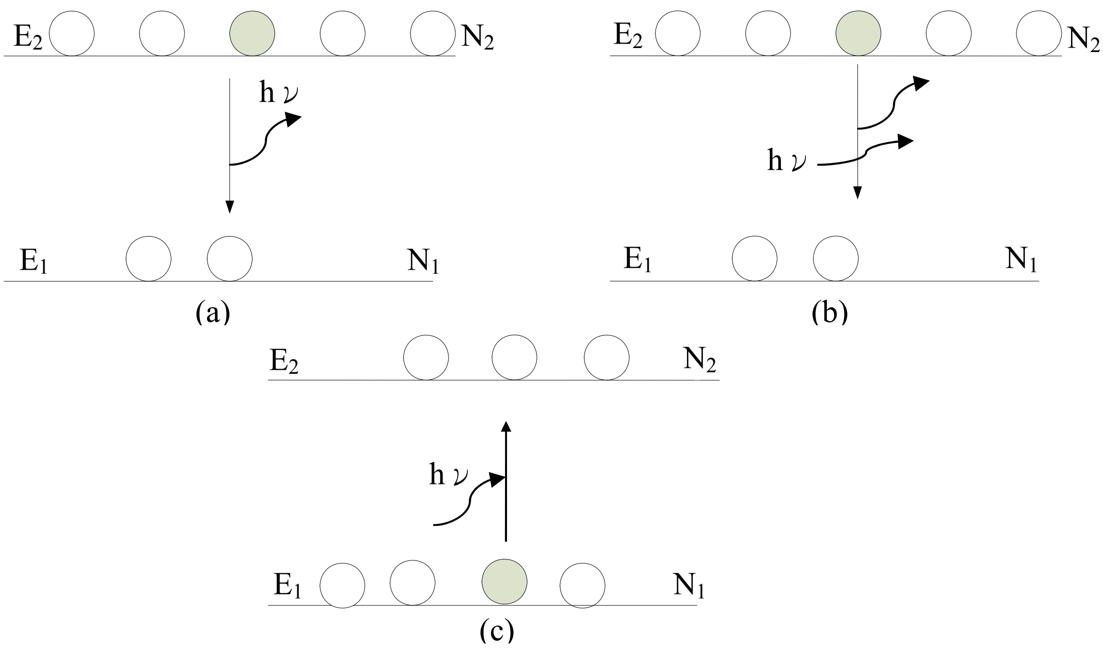

Fig.2-4 Transitions between two energy levels.
(a) Spontaneous emission (b) Stimulated emission
(c) Stimulated absorption

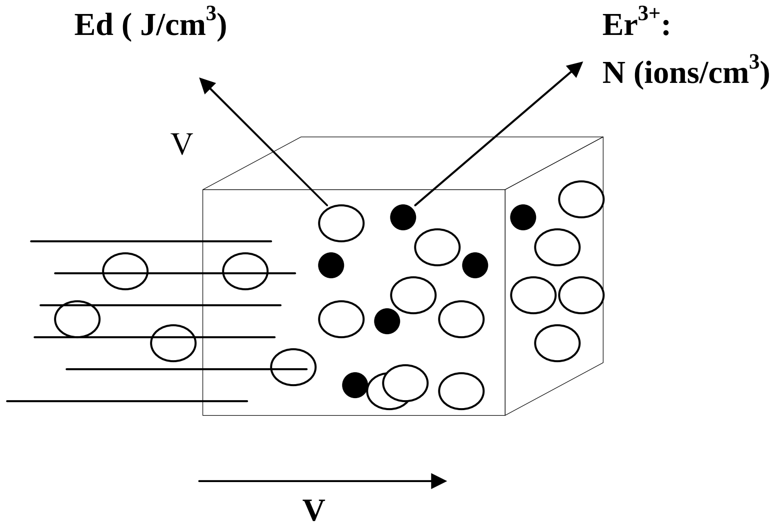

Fig. 2-5 Fictitious moving pill box in a waveguide amplifier

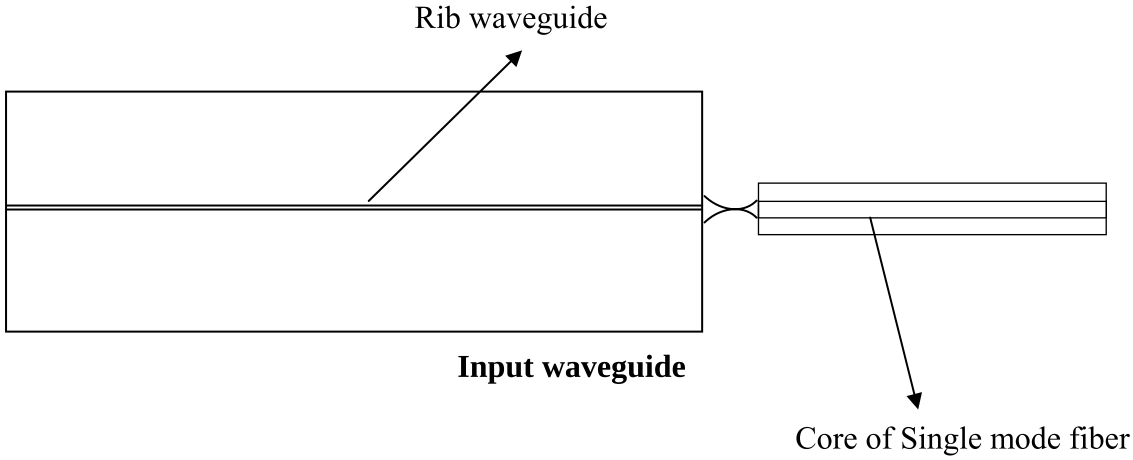

Fig. 2-6 Schematic structure of transmitted beam with single mode fiber into input single waveguide.

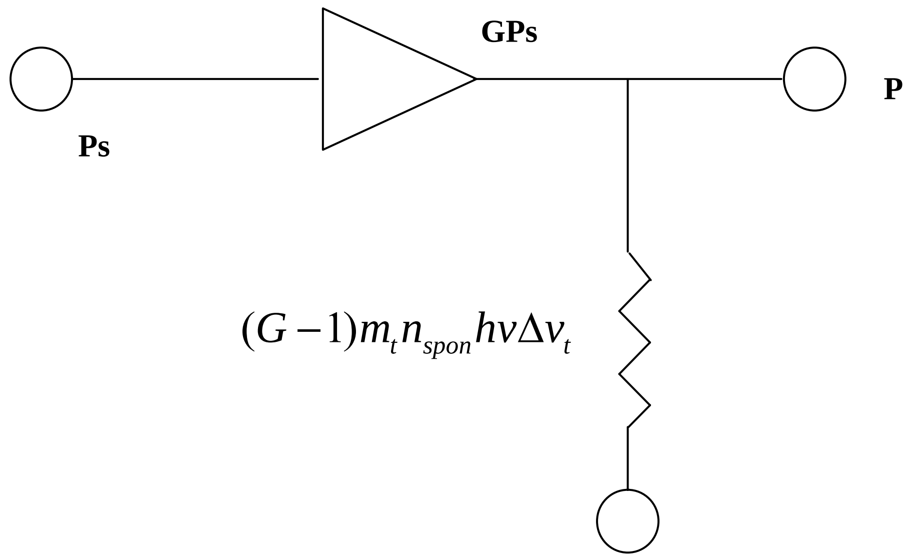

Fig. 2-7 Picture of equivalent of an optical amplifier

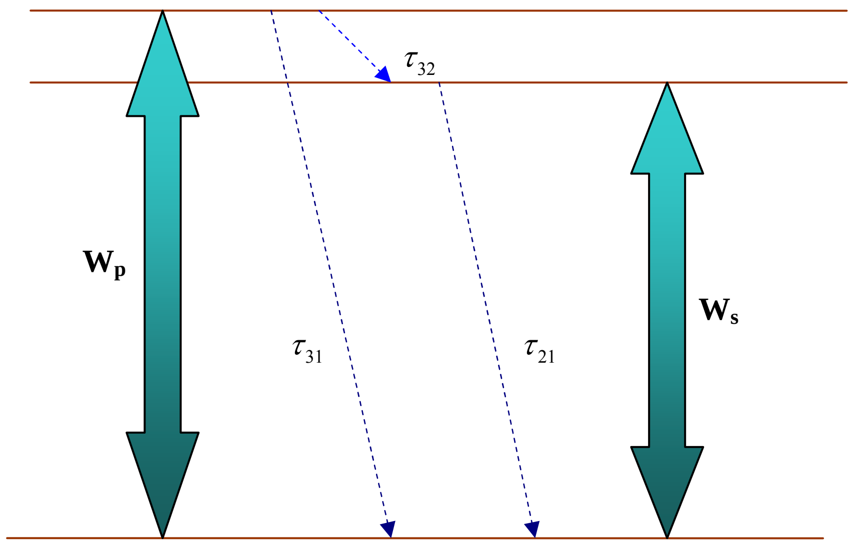

Fig. 2-8 Three level model for Er^{3+} doped waveguide amplifier

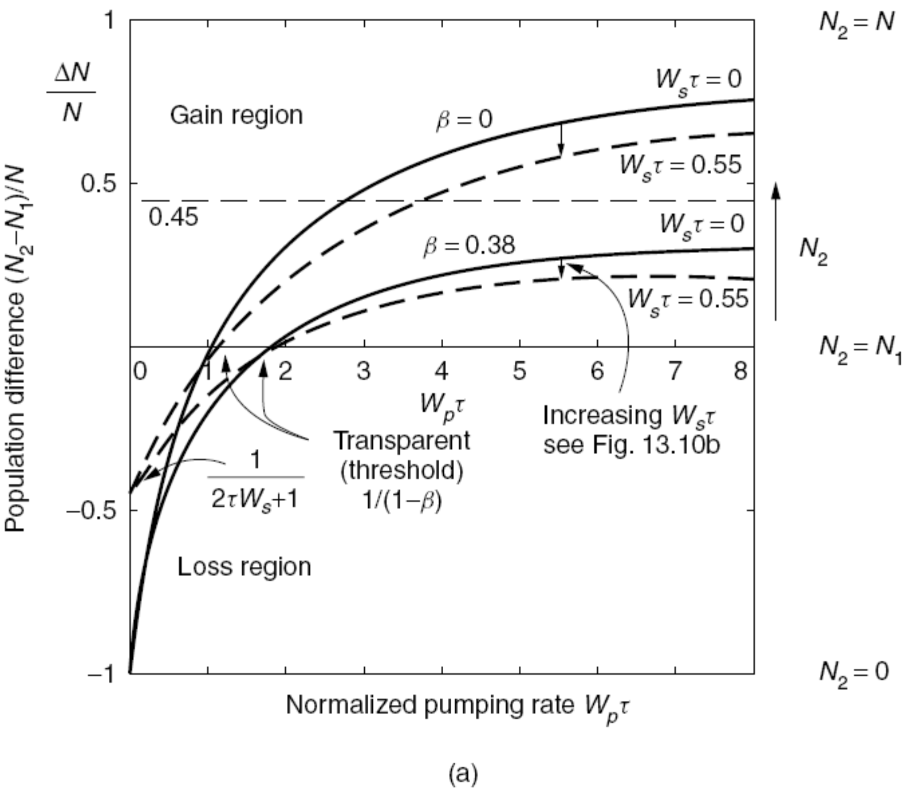

Fig. 2-9 Population difference versus normalized pumping rate with signal power and β as parameters [71].

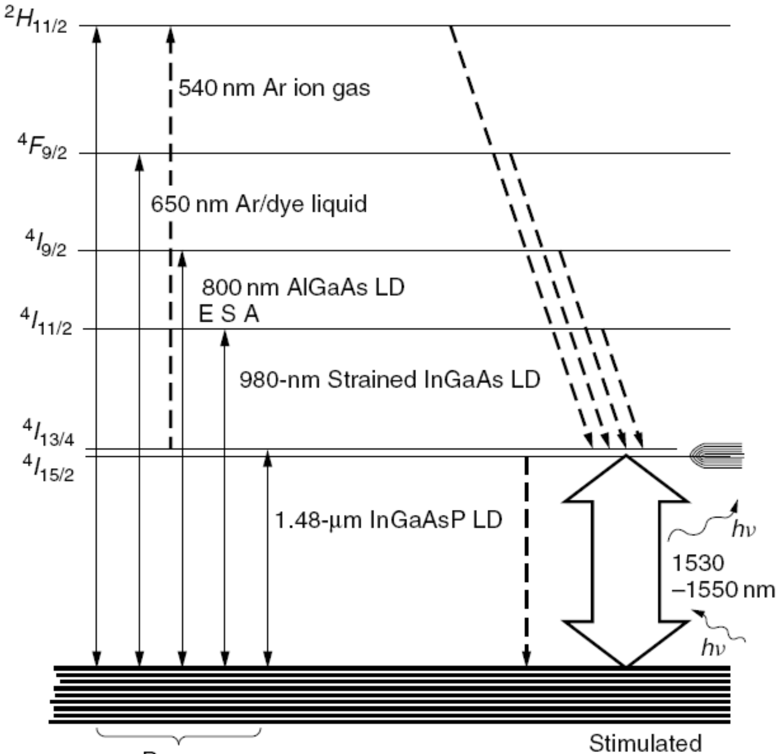

Fig. 2-10 Energy level diagram of Er^{3+} [73]

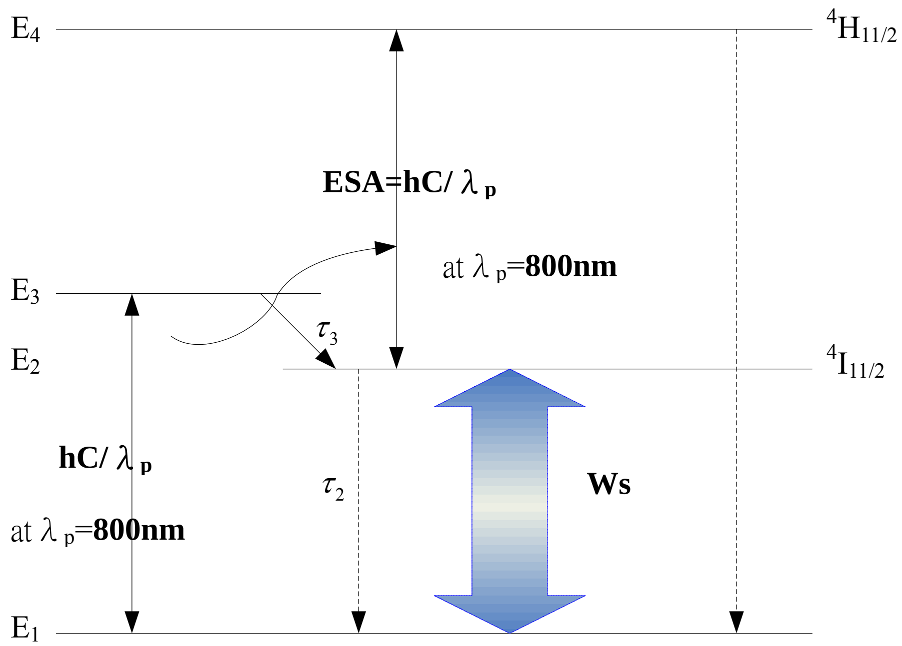

Fig. 2-11 Excited state absorption (ESA)

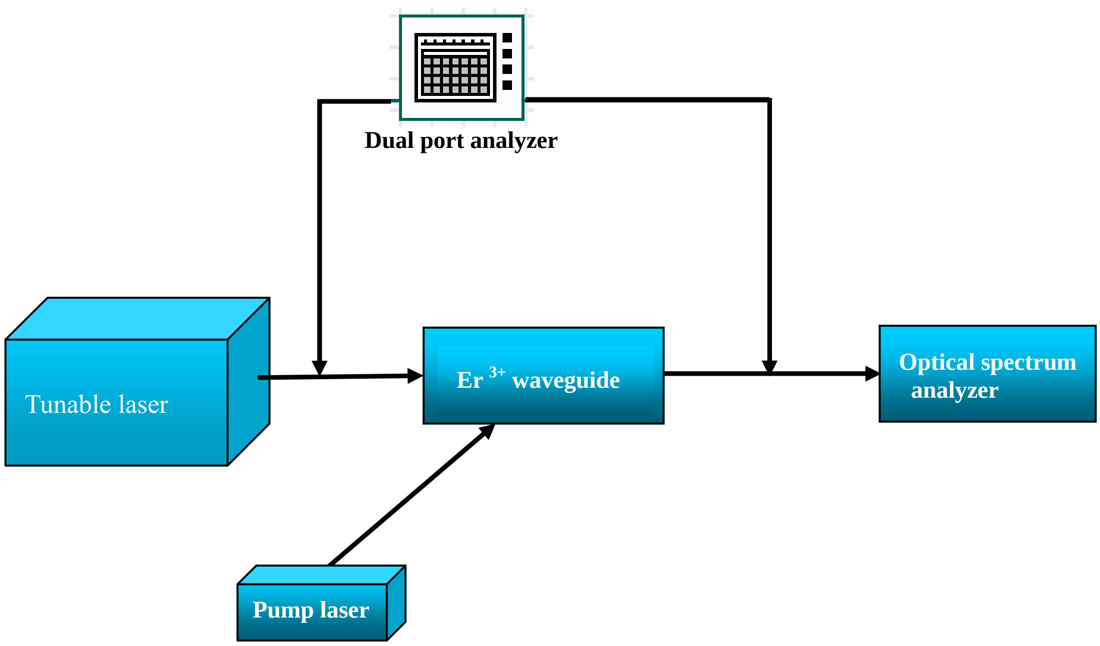

Fig. 2-12 Schematic diagram of single channel Erbium doped waveguide amplifier

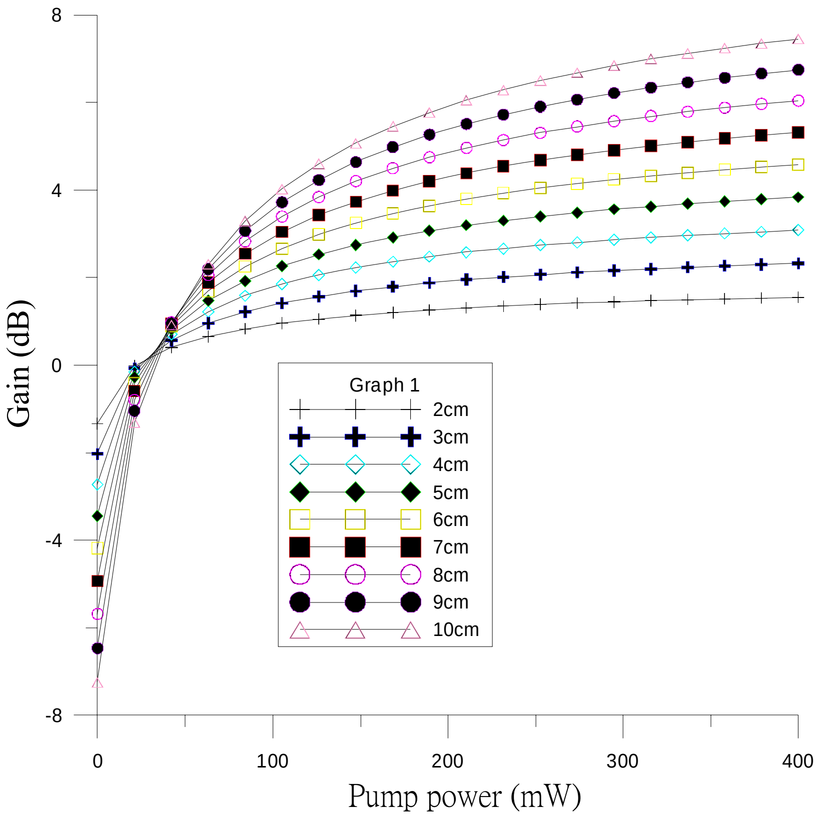

Fig. 2-13 Relationship of Gain and Pump power of different length waveguide amplifier from 0.02m to 0.1m

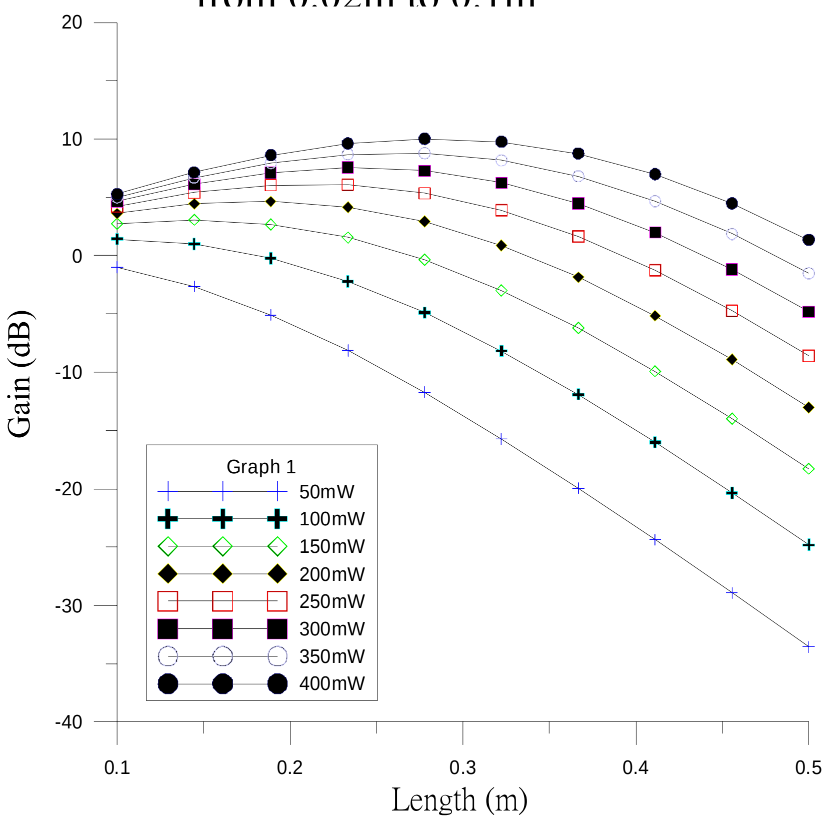

Fig. 2-14 Relationship of Gain and Pump power of different length waveguide amplifier from 0.1m to 0.5m

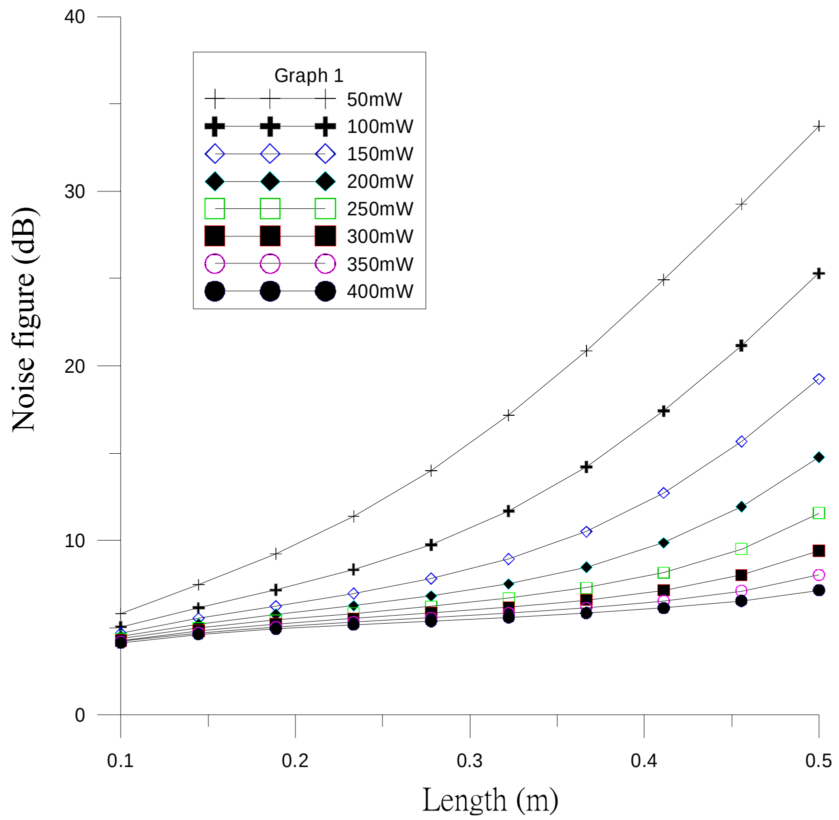

Fig. 2-15 Relationship of Noise figure and Pump power of different length waveguide amplifier from 0.1m to 0.5m