# Chapter 5

# Summaries and Directions for Future Research

In this chapter, we briefly summarized the study reported in this dissertation and give some potential directions for future research.

## 5.1 Summaries

In this study, the optimization algorithms for *deductive games* and related hard-combinatorial optimization (CO) problems are considered. We have investigated a broad variety of algorithms, which include *complete, approximate, deterministic, probabilistic, sequential, and parallel* methods. A series of novel and systematic optimization algorithms are developed for the problems. We conclude the proposed algorithms and the achieved results as follows:

*The graph-partition approach (GP)* has been successfully applied to solve 2×n AB games and 2×n Mastermind games. With this novel approach, we find some symmetric, isomorphic, and recursive structures in the game-guessing process. Therefore, we obtain the optimal strategies for the 2×n games in both the worst and the expected cases.

*The k-way-branching approximate (KWB) and pigeonhole-principle-based backtracking (PPBB) approaches* are proposed for solving deductive games. By using these algorithms, we can obtain an optimal strategy for the 4×10 AB game in the worst case and a near-optimal strategy for the game in the expected case.

A new *elitism-based evolutionary algorithm (EBEA)* has been developed and been successfully applied to the popular NP-complete problem: minimization of Binary Decision

Diagrams (BDDs). EBEA provides an effective termination mechanism, called *the stable function*, for evolutionary algorithms. Some excellent properties are derived from the mechanism. This not only helps to efficiently choose an appropriate termination criterion for a system, but also reduces the run time for redundant generations in the evolutionary process. Experimental results shows that EBEA is robust compared to the latest published BDD minimization algorithms.

Parallel structures for solving hard-CO problems are investigated and developed in DEBEA. Taking advantage of the cooperation of processors as well as the flexible reconfiguration and high communication rate of PC-cluster, DEBEA has an excellent ability to find the optimal results that are not easily discovered by EBEA. DEBEA has improved on previously best-known published results for almost all benchmark circuits in LGsynth91.

## 5.2 Future Research

Although in this study we have developed several optimization algorithms and obtained (sub-)optimal results for deductive games and related combinatorial problems, there is still much room to extend and improve them.

We have derived the optimal strategies for 2×n games. A natural generalization is to explore the techniques for m×n Mastermind games, m≥3. However, our techniques are not easily extensible to the m×n Mastermind games due to the complexity of this problem. This problem remains open. On the other hand, deciding what complexity class the m×n (or just 3×n) games belong to is also an important topic to study.

The KWB algorithm cooperating with the concept of HCG could be applied to solve other combinatorial optimization problems due to its ability to approximate optimal results. However, the run time may grow exponentially if the parameter k is increased. An efficient approximate method is required to overcome the obstacles of run time explosion.

Optimization problems for deductive games with errors, such as Ulam's games [Ula76], are related to error-correcting codes in coding theory. A large class of open problems in this domain were surveyed and suggested in [Pel02]. In addition, Mastermind and AB games with faults are also interesting research topics.

Evolutionary algorithms (EAs) are stochastic search methods that have been applied successfully in many search, optimization, and machine learning problems. EBEA is a robust model to solve hard-CO problems provided that the domain heuristic for the problem at hand is strong enough. To apply EBEA to solve other hard-CO problems, how to choose appropriate parameters for the stable function is an important issue. Some machine learning techniques, such as using *artificial neural networks* to learn proper parameters for EBEA, may provide solutions to the issue. Adaptive-EBEA will be an excellent candidate for solving hard-CO problems.

Decentralizing and parallelizing an EA have received more and more attention in the past two decades, due to the excellent ability to find globally optimal results. Some of the most promising research topics in the field of parallel EAs were addressed in [Alb02b]. We list the following two issues for future research.

- Apply DEBEA to deal with *dynamic function optimization problems* whose optimal solutions vary in time, such as controlling an elevator or the traffic light system. Due to the nature of the selection techniques used in EA, all we have to do is to change the fitness function to react a change in the dynamic environment.
- Develop a hybrid model, which consists of EAs and branch-and-bound algorithms, to efficiently obtain exactly optimal results.

The optimization algorithms developed and the results obtained in this study could also be applied to many state-of-the-art CO problems, such as coding problems, circuit testing, additive search problem, online models with equivalent queries and differential cryptanalysis as well as other combinatorial optimization problems. We hope that the methodologies developed in this dissertation could be applied to solve other related problems in the future.